

Feasibility of Determining Diffuse Ultra-High Energy Cosmic Neutrino Flavor Ratio through ARA Neutrino Observatory

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Abstract

The flavor composition of ultra-high energy cosmic neutrinos (UHECN) carries precious information about the physical properties of their sources, the nature of neutrino oscillations and possible exotic physics involved during the propagation. Since UHECN with different incoming directions would propagate through different amounts of matter in Earth and since different flavors of charged leptons produced in the neutrino-nucleon charged-current (CC) interaction would have different energy-loss behaviors in the medium, measurement of the angular distribution of incoming events by a neutrino observatory can in principle be employed to help determine the UHECN flavor ratio. In this paper we report on our investigation of the feasibility of such an attempt. Simulations were performed, where the detector configuration was based on the proposed Askaryan Radio Array (ARA) Observatory at the South Pole, to investigate the expected event-direction distribution for each flavor. Assuming ν_μ - ν_τ symmetry and invoking the standard oscillation and the neutrino decay scenarios, the probability distribution functions (PDF) of the event directions are utilized to extract the flavor ratio of cosmogenic neutrinos on Earth. The simulation results are summarized in terms of the probability of flavor ratio extraction and resolution as functions of the number of observed events and the angular resolution of neutrino directions. We show that it is feasible to constrain the UHECN flavor ratio using the proposed ARA Observatory.

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I. INTRODUCTION

The observed energy spectrum of cosmic rays has been extended to beyond 10^{20} eV [1, 2], but little is known about their origins and acceleration mechanism, which are important questions in astrophysics [3]. Ultra-high energy cosmic rays (UHECRs) are thought to be of extragalactic origin, such as being produced by active galactic nuclei (AGNs) or gamma-ray bursts (GRBs) [4]. Such UHECRs can generate ultra-high energy ($E > 10^{17}$ eV) neutrinos via photo-pion production or proton-proton interaction:

$$\begin{aligned} p + \gamma &\rightarrow \Delta^+ \rightarrow n + \pi^+, \\ p + p &\rightarrow \pi^+ \pi^- \pi^0, \end{aligned}$$

and the subsequent decays of charged pion and muon, e.g.,

$$\pi^+ \rightarrow \nu_\mu + \mu^+ \rightarrow \nu_\mu + \bar{\nu}_\mu + \nu_e + e^+,$$

where the targets can be the intergalactic medium near the astrophysical sources [5, 6] (see Ref. [7] for a review), or the cosmic microwave background (CMB) photons (the Greisen-Zatsepin-Kuzmin process) [8, 9]. Neutrinos originating from the GZK process are known as the cosmogenic neutrinos (or GZK neutrinos), which are guaranteed to exist based on the fact that both initial-state particles, i.e., the UHECR and the CMB photon, have been observed and that the notion is consistent with the observed GZK cutoff in the cosmic ray spectrum [10]. Since the production of ultra high energy cosmic neutrinos (UHECNs) are tightly connected with UHECRs, such neutrino spectrum can help to resolve the puzzles of cosmic rays such as their composition [11, 12], the energy spectrum at the sources, and the cosmological evolution of the sources [13, 14].

Besides the overall spectrum, the relative flux ratio between different neutrino flavors, or briefly, the flavor ratio, can also provide information about the physical properties of UHECR sources. For example, the transition of flavor ratio at the source from $f_e^S : f_\mu^S : f_\tau^S = 1 : 2 : 0$ (pion source) to $0 : 1 : 0$ (muon-damped source) with increasing energies due to synchrotron energy loss of muons can be used to constrain the strength of cosmic magnetic field [15–17] (neutrinos and antineutrinos are counted together because they are hard to be discriminated in the UHE neutrino detection). Furthermore, during the propagation from the source to the Earth, the flavor composition of a neutrino would oscillate [18], and may even be altered by

some new physics beyond the Standard Model, such as the neutrino decay [19–23], pseudo-Dirac states of neutrinos [24], sterile neutrinos with tiny mass differences [25], the violation of CPT or Lorentz invariance [20, 23, 26], and the quantum decoherence [23, 26, 27] (see Ref. [28] for a review). With extremely high energies and long traveling distances ($> 10\text{Mpc}$), the flavor ratio of UHECNs can also help constrain neutrino oscillation parameters [29–34] and probe exotic physics in the parameter regime inaccessible on Earth.¹

To detect UHECNs, enormous amount of matter is required for the target due to their low flux and tiny interaction cross section. There are four major detection strategies depending on either neutrinos interact with nucleons via the neutral current (NC) interaction, $\nu_l + N \rightarrow \nu_l + X$, or via the charged current (CC) interaction, $\nu_l + N \rightarrow l^- + X$, where l stands for lepton and X for hadronic debris that will develop into hadronic showers. The first approach is to observe the optical Cherenkov lights emitted by secondary charged particles and showers by an array of optical sensors (e.g. photomultiplier tubes) deployed deep in the medium, e.g. under-ice arrays such as AMANDA [35] and IceCube [36] at the South Pole, and Baikal, ANTARES, NESTOR, NEMO, KM3NET [37] underwater. The second one is to detect horizontal or Earth-skimming neutrino-induced air showers, such as Pierre Auger Observatory [38] and HiRes [39]. The third approach is to detect acoustic waves generated by the showers, which is still in the R&D stage [40]. The last and a very promising one is to observe the radio Cherenkov emission from the neutrino-induced showers in dense media through the Askaryan effect [41]. Showers propagating in dense media would develop about 20 % of excess negative charges and would emit Cherenkov radiation, which is coherent in the radio frequencies up to a few GHz due to the compact shower size. This effect has been verified in a series of beam experiments [42]. The radiated power in the coherent regime is proportional to the square of net charges, which is roughly proportional to the shower energy, making this technique especially sensitive to UHE showers and thus UHECNs. Another advantage of this approach is the long radio attenuation length in some natural media, e.g. the Polar ice and salt, with lengths typically of order of 0.1 to 1 km, and therefore detectors are able to monitor large target volume and achieve greater sensitivity. Observatories of this type are: the FORTE satellite [43] looking for neutrino signals from the Greenland ice; the balloon-borne antenna array ANITA [44] overlooking the Antarctic ice; and radio telescopes looking for signals from the lunar regolith, e.g. GLUE [45] and

¹ In fact, most of these references consider neutrinos with energy $> \text{PeV}$.

LUNASKA [46]. There are also attempts to deploy antennas inside the target media in order to lower the threshold energy (to about 10^{17} eV), e.g. under-ice antenna arrays RICE [47], ARA [48], and ARIANNA [49] in the Antarctic ice; and SalSA [50] in salt dome.

It is impossible to distinguish between neutrino flavors from the NC interactions because their only products are hadronic showers. On the other hand, the charged leptons produced in the CC interactions have different energy-loss characteristics in the medium, which provides an opportunity to identify the flavor. Optical Cherenkov neutrino telescope, e.g. IceCube [36], is able to identify the flavors by event topologies. The muon from the ν_μ CC interaction leaves a track; whereas showers from all flavor's NC events and ν_e 's CC events lead to localized trigger patterns; and there are double-bang and lollipop events unique to ν_τ induced by the τ decays. Beacom et al. [51] proposed a method to deduce the neutrino flavor ratio from the measured events of different types, and its feasibility has been widely investigated (e.g. [21, 29, 30, 32, 52]). However, the instrumented volume, currently of cubic kilometer scale for the largest, limits the event rate for UHE neutrinos, which renders it challenging to distinguish between ν_μ and ν_τ events at UHEs as the decay length of τ lepton exceeds the detector size [53].

For radio Cherenkov telescopes such as ARA, the situation is a somewhat different. Though this approach cannot detect the track by a single charged particle, the ν_μ and ν_τ CC events can in principle be separated through the amount of energy deposited by leptons into electromagnetic and hadronic showers [53]. It has also been pointed out that different types of showers at UHEs can be distinguished according to their elongation by the Landau-Pomeranchuk-Migdal (LPM) effect [54, 55]. In addition, ν_e CC events can generate mixed showers of both types and should have its own characteristic signal feature [56]. The feasibility of this method has been investigated in SalSA [57].

Apart from the event signatures, the direction distribution of neutrino events also manifest themselves in the the energy-loss properties of leptons since neutrinos from different directions propagate through different amounts of matter in the Earth. For example, ν_τ can undergo the regeneration process ($\nu_\tau \rightarrow \tau \rightarrow \nu_\tau$) without being absorbed due to the τ decay [53, 58], and as a result can exhibit a higher flux in the up-going directions. Therefore the neutrino distribution can be a useful tool for measuring the flavor ratio and should be applicable to any detector with sufficient angular resolution of event direction. A similar idea that takes advantage of the event direction distribution has been proposed to constrain

neutrino-nucleon cross sections [59].

In this paper we focus on the cosmogenic neutrinos and consider three expected flavor ratios when they arrive at the Earth’s surface, that is, $1 : 1 : 1$ expected in the standard oscillation scenario [18]; and $6:1:1$ as well as $0:1:1$ ratios predicted in the neutrino decay models [19] with normal and inverted neutrino mass hierarchy, respectively. The sum of ratios is normalized to unity in the following sections. With the detector configuration based on ARA [48] currently under construction at the South Pole, we demonstrate the feasibility of extracting the flavor ratio from the event direction distribution, while the inference on the flavor ratio at the source is beyond our scope.

In the next section we present the simulation setup, and derive the expected event direction distribution for each flavor in Section III. The procedures for hypothetical experiments and the extraction of flavor ratios from the direction distribution of pseudo-data are described in Section IV A and IV B. The successful probability of this method and the flavor ratio resolution as functions of the number of observed events and angular resolution of neutrino direction are reported in Section IV C.

II. SIMULATION SETUP

Our simulation code is built by integrating existing packages that consists of two parts. The first part is the propagation of the neutrinos and the secondary charged leptons from the Earth’s surface, where the primary UHECN is generated, to the vicinity of the detector. The second part is the simulation of event detection, including the conversion of particle energy losses to showers, the development of Cherenkov radiations from showers, the propagation of Cherenkov radiations to detectors, and the calculation of detector responses. The planned configuration of ARA [48] and the ice properties at the South Pole are adopted as the setting in our simulations.

A. Neutrino Generation and Propagation Using MMC

The Muon Monte Carlo (MMC) package [60] is employed to generate neutrinos and propagate all types of neutrinos and secondary charged leptons. In MMC, interaction cross sections of neutrinos are evaluated based on Ref. [61] with CTEQ6 parton distribution functions [62].

For charged leptons, energy losses via ionization, pair production, bremsstrahlung, photonuclear interaction, and decay are taken into account, so is $\nu_\tau \rightarrow \tau \rightarrow \nu_\tau$ regeneration. The Kelner-Kokoulin-Petrukhin (KKP) [63] and Bezrukov-Bugaev (BB) [64] parameterizations are chosen for cross section calculations of bremsstrahlung and photonuclear interaction, respectively. We do not propagate secondary electrons and regard them as losing all of their energies to shower developments within a short distance once they are generated.

Monoenergetic neutrinos are generated isotropically at the Earth's surface and start their propagation to the detection volume. The Earth model provided in MMC simulation code is used, where the density is calculated based on the Preliminary Reference Earth Model (PREM) [65] while the composition as well as the topography on the Earth's crust are not considered. The detection volume, which is the South Polar ice sheet in the vicinity of ARA, is approximated by a cylindrical ice volume, centered at 1 km below the ice surface with a radius of 8 km and a height of 2 km. In the propagation, neutrinos and secondary charged leptons are traced until they are either absorbed by the Earth or exiting the detection volume; energy losses greater than 1 PeV are treated stochastically. Only those neutrino events traversing the detection volume with at least one interaction and with energy loss exceeding 1 PeV are reserved for the event detection. The cutoff energy is chosen based on the consideration that the radio Cherenkov signals emitted by showers below this energy would not be strong enough to trigger the detector efficiently.

B. Neutrino Event Detection Using SADE

The Simulation of Askaryan Detection and Events (SADE) package [66] is used for simulating neutrino detection. Neutrino events recorded in the previous step, as described in Section II A, are processed individually. Every energy loss exceeding 1 PeV within the detection volume is converted into showers of corresponding types. Hadronic products of neutrino CC and NC interactions, τ decay, and photonuclear interactions are turned into hadronic showers, while secondary electron, pair production, and bremsstrahlung are turned into electromagnetic showers. In general, a neutrino event can generate multiple showers in the detection volume. For example, a ν_e event having CC interaction would generate a hadronic shower and an electromagnetic shower, whereas a ν_μ or a ν_τ event may have several to even hundreds of showers produced by the secondary μ or τ lepton, respectively.

In SADE, shower characteristics and the frequency spectrum of Cherenkov radiation are calculated with analytic formulae, where the latter is primarily based on Ref. [67] but with a little bit modification of parametrization to account for the decoherence due to the longitudinal and the lateral spreads of shower (see Appendix A for details).

A ray-tracing routine then finds the paths of both the direct and the reflected (due to the ice-air interface at the surface) rays connecting each shower to each antenna in the ice whose index of refraction varies with depth. The flight time, the radiation spectrum taken into account the frequency-dependent attenuation along the path as well as the polarization at the antenna are calculated for each ray. The spectrum is then Fourier-transformed to the electric field received by the antenna in the time domain accordingly.

Ice properties such as the index of refraction [68], the temperature [69], and the radio attenuation length [48, 70] are based on the results of *in situ* measurements at the South Pole. The index of refraction n and the ice temperature T (in °C) depend only on the depth (in km), $|z|$,

$$n(z) = 1.78 - (1.78 - 1.35) \exp(-13.2|z|), \quad (1)$$

$$T(z) = -51.5 - 0.45319|z| + 5.822|z|^2. \quad (2)$$

The attenuation length in turn depends on the ice temperature as well as the radiation frequency, and is plotted in Fig. 1. Note that the rising ice temperature with increasing depth renders the attenuation length shorter and thus suppresses the detectability of signals originating from the bottom part of ice. The birefringence of South Polar ice, which is the polarization dependence of the wave speed and the attenuation due to the crystal anisotropy and orientation of ice, is not considered here. It is reported [71] that the birefringence is observed at the bottom half of the ice sheet and will reduce about 5% of the neutrino detection volume.

The detector based on the planned configuration of ARA [48] is a hexagonal array of 37 antenna stations arranged in a triangular grid with 2 km spacing. The array covers a total area of about 100 km². Each station is an autonomously operating cluster of eight vertically polarized (Vpol) and eight horizontally polarized (Hpol) antennas evenly deployed on four vertical strings with each string placed on one vertex of a square. Each string is at a maximum depth of 200 m and is loaded with two antenna pairs, where each antenna pair contains a Vpol antenna and an Hpol antenna. The values of parameters for the detector

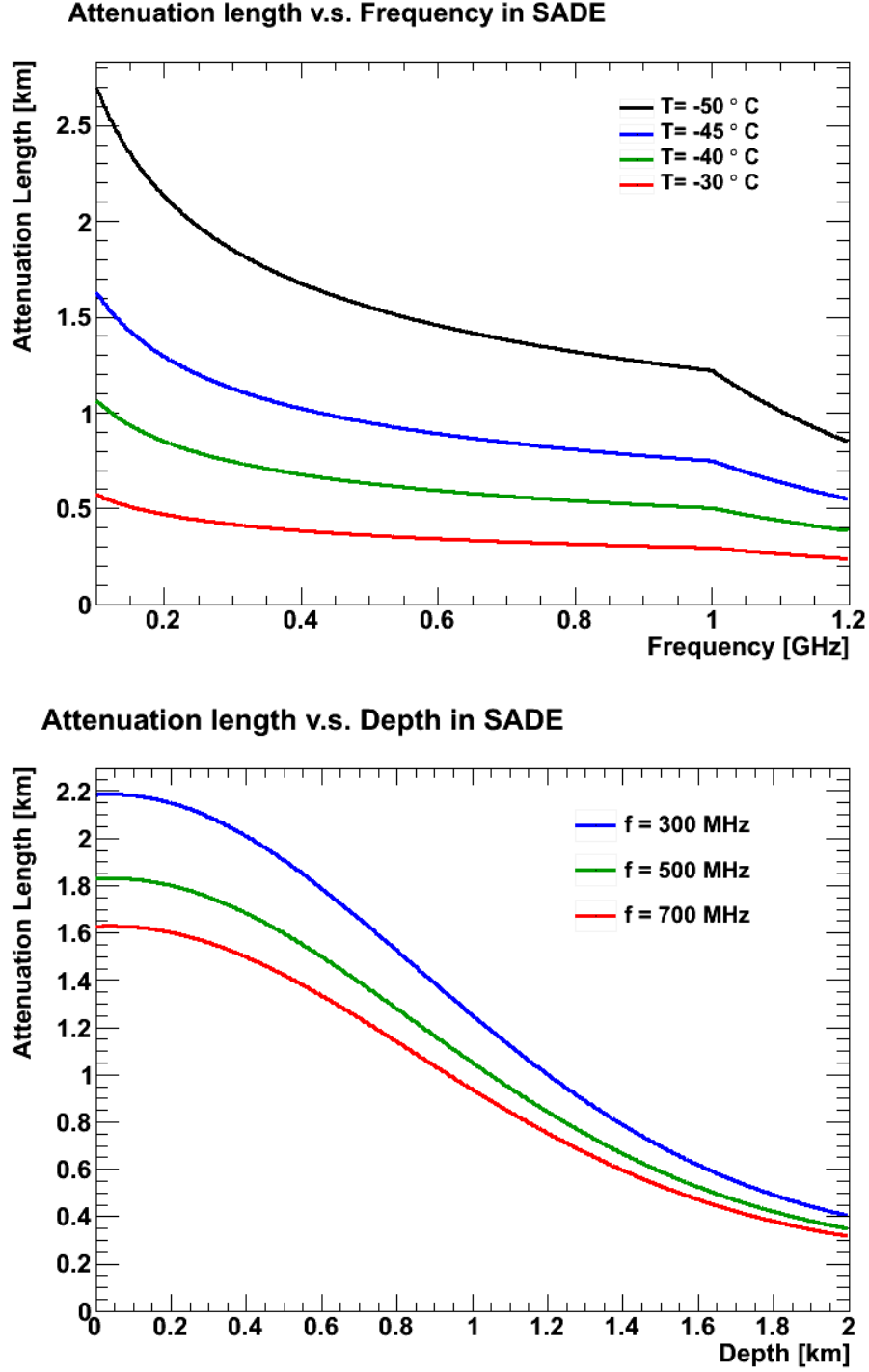


FIG. 1. Attenuation length used in SADE [?] as a function of the radiation frequency at different temperatures (-30°C , -40°C , -45°C and -50°C , top panel) and as a function of the depth at different frequencies (300 MHz, 500 MHz and 700 MHz, bottom panel), respectively.

settings are listed in Table I.

Given the incident electric field \vec{E} at an antenna, the received signal voltage of the antenna V_{signal} is

$$V_{\text{signal}} = \frac{1}{2} \vec{E} \cdot \vec{h}_{\text{eff}}, \quad (3)$$

with

$$|\vec{h}_{\text{eff}}| = 2 \sqrt{\frac{A_{\text{eff}} Z_{\text{ant}}}{n Z_0}}, \quad (4)$$

$$A_{\text{eff}} = \frac{G c^2}{4 \pi f^2}, \quad (5)$$

where \vec{h}_{eff} and A_{eff} are the effective height and the effective area of the antenna, respectively; Z_{ant} the antenna impedance, $Z_0 \simeq 377 \Omega$ the impedance of free space, n the index of refraction of surrounding medium, G the antenna gain, and c the speed of light in vacuum. The direction of effective height for Vpol antennas is in the vertical direction \hat{z} , while it is in the azimuthal direction $\hat{\phi}$ for the Hpol. In the simulation, the frequency response of the antenna is assumed to be a single perfect passband, and the effective height is approximated by a single value evaluated at the central frequency of the passband. For each antenna, signals coming from different showers are summed in the time domain. We neglect the contribution from the reflected signals because they would have sufficient time delays and would suffer more attenuation than the direct ones due to longer path length and are more difficult to be reconstructed after passing through the less compact snow near the surface (firn) where the index of refraction changes rapidly (see Eqn. 1).

The root mean square (RMS) thermal noise voltage of an antenna V_{rms} is defined as

$$V_{\text{rms}} = \sqrt{k_B T_{\text{sys}} Z_{\text{ant}} B}, \quad (6)$$

where k_B is Boltzmann's constant, T_{sys} the system noise temperature of the receiving antenna system, f the radiation frequency and B the frequency bandwidth of the antenna. The values of antenna parameters used in the simulation are summarized in Table I.

The trigger conditions for a detected event require that *i*) the received voltage of a triggered antenna should exceed three times of its RMS noise voltage (i.e., $V_{\text{signal}} \geq 3V_{\text{rms}}$); *ii*) at least five out of sixteen antennas in a station are triggered; and *iii*) at least one station is triggered.

Monoenergetic neutrinos with initial energies $\log_{10}(E_\nu/\text{eV}) = 17, 17.5, 18, 18.5, 19$ and 19.5 are generated separately, and the numbers of events generated and detected are listed in Table II (see Appendix).

TABLE I. Parameters for the detector configuration and the antennas, and their values used in the simulation.

Parameter (unit)	Value
Station spacing (km)	2
Radius of string (m)	10
Number of strings per station	4
Separation between paired antennas (m)	5
Vertical spacing between antenna pairs (m)	20
Maximum antenna depth (m)	200
Number of Vpol antennas per station	8
Number of Hpol antennas per station	8
Vertical antenna configuration	V,H above V,H
Vpol antenna bandwidth: B_V (MHz)	150-850
Hpol antenna bandwidth: B_H (MHz)	200-850
Antenna impedance: Z_{ant} (Ω)	50
Antenna gain: G	1.64
Effective height of Vpol antenna: $ \vec{h}_{\text{eff},V} $ (cm)	11.8
Effective height of Hpol antenna: $ \vec{h}_{\text{eff},H} $ (cm)	11.3
System noise temperature of antenna: T_{sys} (K)	300
RMS noise voltage of Vpol: $V_{\text{rms},V}$ (V)	1.20×10^5
RMS noise voltage of Hpol: $V_{\text{rms},H}$ (V)	1.16×10^{-5}
Antenna trigger threshold (V_{rms})	3
Station trigger threshold (antennas)	5

III. ANGULAR DISTRIBUTION OF INCOMING NEUTRINOS

To acquire the expected direction distribution of detected neutrino events with initial energy E_ν and flavor α , $D_\alpha(E_\nu, \cos \theta)$, the following information are needed:

- a) the flux of isotropic cosmogenic neutrinos for all flavors, $\Phi_\nu(E_\nu)$, as well as the incident flux ratio among three flavors at the Earth's surface, $f_e^E : f_\mu^E : f_\tau^E$, with $\sum_\alpha f_\alpha^E = 1$;
- b) the interaction probability $P_{\text{int},\alpha}(E_\nu, \cos \theta)$, which is defined as the probability that a neutrino or its secondary lepton traverses the Earth in the zenith direction $\cos \theta$ without being stopped and interacts (with at least one energy loss exceeding 1 PeV) inside the detection volume, to account for the propagation effect; and
- c) the detection efficiency to the subsequent shower(s) generated by the neutrino event interacting inside the detection volume, $\epsilon_{\text{det},\alpha}(E_\nu, \cos \theta)$. That is,

$$D_\alpha(E_\nu, \cos \theta) = \frac{1}{\mathbb{N}} f_\alpha^E \Phi_\nu(E_\nu) P_{\text{int},\alpha}(E_\nu, \cos \theta) \times \epsilon_{\text{det},\alpha}(E_\nu, \cos \theta), \quad (7)$$

with the normalization factor

$$\mathbb{N} = \sum_\alpha \int dE_\nu \int_{-1}^1 d\cos \theta f_\alpha^E \Phi_\nu(E_\nu) \times P_{\text{int},\alpha}(E_\nu, \cos \theta) \epsilon_{\text{det},\alpha}(E_\nu, \cos \theta), \quad (8)$$

where the distribution has been normalized as a probability distribution independent of the total event rate, $\alpha = e, \mu, \tau$, the superscript E indicates quantities on the Earth, and θ is the angle between the local vertical axis of the detector (\hat{z}) and the direction of neutrino momentum. Hence, events with negative $\cos \theta$ are down-going, whereas those with positive values are up-going. The neutrino energy range considered in this article is $\log_{10}(E_\nu/\text{eV}) = 16.75\text{--}19.75$.

A. Flux and Flavor Ratio on Earth

The cosmogenic neutrino fluxes for all flavors, $\Phi_\nu(E_\nu)$, have been theoretically predicted in Refs. [14, 72, 73] (see Fig. 2). In the following analysis, the neutrino flux from Ref. [72] (red solid curve; hereafter, ESS) is assumed. Note that it is the spectral shape that affects the event direction distribution instead of the overall flux normalization. Therefore one

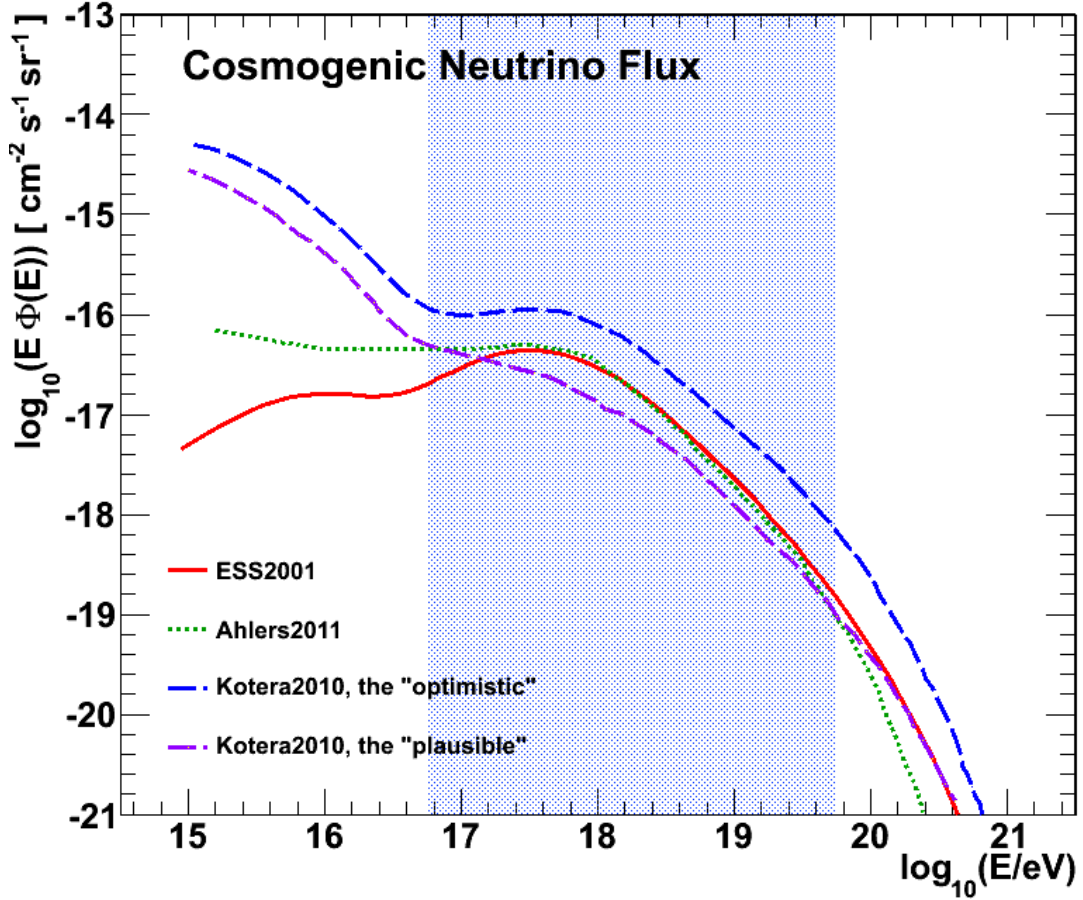


FIG. 2. Differential cosmogenic neutrino fluxes predicted by [72] (ESS, red solid curve), [73] (green dashed), the optimistic (purple dashed) and the plausible (blue dashed) scenarios in [14]. The shaded region indicates the neutrino energy range considered in the analysis, $\log_{10}(E_\nu/\text{eV}) = 16.75\text{--}19.75$.

expects that neutrino fluxes predicted in [72], [73] (green dashed curve), and the optimistic scenario in [14] (purple dashed curve) should yield similar distributions. The flux predicted in the plausible scenario in [14] (blue dashed curve) has a steeper spectrum than others, and we will present its results later in Sec. IV.

The flavor ratio of neutrinos arriving at the Earth's surface adopted in our analysis is $f_e^E : f_\mu^E : f_\tau^E = 1/3 : 1/3 : 1/3$, as expected in the standard oscillation scenario [18]; $0.75 : 0.125 : 0.125$ and $0 : 0.5 : 0.5$ predicted in the neutrino decay scenarios with normal and inverted neutrino mass hierarchy, respectively [19]. Throughout this paper these flavor ratios are assumed to be energy-independent over the considered neutrino energy range.

B. Interaction Probability

To obtain the interaction probability P_{int} from the simulation results, we first divide the zenith angle of neutrinos $\cos\theta$ into 100 bins with a width of 0.02. The probability at each bin is defined as the ratio of the number of survival events inside the detection volume to the number of initial incoming events at the Earth's surface. The probability does not depend on the azimuthal angle of the neutrino due to the axial symmetry of our Earth model and detection volume. The probabilities $P_{\text{int},\alpha}(E_\nu, \cos\theta)$ for initial neutrino energies $E_\nu = 10^{17}$ eV, 10^{18} eV and 10^{19} eV are shown in Fig. 3.

For different flavors of neutrinos with the same initial energy, the interaction probabilities are about the same as $\cos\theta$ approaches -1 where neutrinos impinging directly downward into the detection volume, whose size is much smaller than the neutrino interaction length (about several hundred km [74]). Thus the probability is approximately equal to the size of the detection volume divided by the neutrino interaction length for energy transfers greater than 1 PeV. The probability for neutrinos impinging downward increases with neutrino energy as the neutrino interaction length decreases.

The probabilities then increase with $\cos\theta$ and reach a maximum near the horizontal direction ($\cos\theta \simeq 0$), where the traveling distance of neutrinos becomes comparable to the interaction length and the detection volume has its maximum span. The probabilities for different flavors diverge due to the difference in the energy-loss property between different flavors of charged leptons produced in the CC interactions. The longer the lepton can propagate, the higher the probability is. Contrary to electrons, which would be stopped immediately after their creation and would develop into electromagnetic showers, EeV μ and τ leptons can on the average propagate distance of order of 10 km before come to a stop [75]. Muons lose their energy mostly via pair production, while tauons via pair production as well as photonuclear interaction. So the probabilities of finding ν_μ and ν_τ are higher than that for ν_e . The interaction probability in these directions also increases with neutrino energies because of the decrease of the neutrino interaction length and the lepton propagation range.

The probability for up-going neutrinos ($\cos\theta > 0$) is suppressed as the neutrino traveling distance becomes longer than the interaction length. The higher the initial neutrino energy, the shorter the interaction length, and hence the distribution terminates at smaller $\cos\theta$.

The Earth attenuates the neutrino both in energy through NC and CC interactions and in number through the stoppage of the charged lepton produced in the CC interaction. As a special case, τ lepton, having a decay length of about $50 \times (E_\tau/\text{PeV})$ m, can transform to ν_τ through decay before losing too much energy [53, 58]. Therefore ν_τ coming from below the horizon would have an apparent larger probability than other two flavors. This is a critical feature for the flavor ratio determination proposed in this article, as we will further elaborate below.

C. Detection Efficiency

The detection efficiency ϵ_{det} defined here depends not only on the nature of Cherenkov radiation, the ice properties and the detector configuration, but also on how neutrinos and secondary leptons deposit their energies in the shower. But for the simplicity of computation, we do not decompose it further into a product of the probability that a neutrino or a secondary lepton in specific direction generates shower(s) of specific energy at specific position, times the detection efficiency to each individual shower.

Similar to the definition of the interaction probability, the detection efficiency is defined as the number of events detected divided by the number of events interacting inside the detection volume. The detection efficiency is hence an averaged quantity over the azimuthal direction and the shower position. The results for different initial neutrino energies and flavors are shown in Fig. 4.

The efficiency increases with neutrino energy simply because shower size is proportional to neutrino energy. The Cherenkov cone generated by neutrinos events with $\cos \theta \simeq -0.7$ travels downward and has an opening angle 56° in ice, so it is less possible to cover the area where the antennas are deployed. This leads to the common cutoff at $\cos \theta \simeq -0.7$ for all neutrino energies. In Fig. 4, we see the fluctuations of the efficiencies for ν_e (top panel) and ν_μ (middle panel) at $\cos \theta \gtrsim 0.1$. This is due to the smallness of the number of events arriving at the detection volume. Such results are therefore not reliable. However one generic feature remains valid; that is, the up-going neutrino events diminish since their energies are severely damped by the Earth.

The detection efficiency for ν_e is the highest among the three flavors, because once CC interaction occurs all the neutrino energy is released into showers and strong signals are

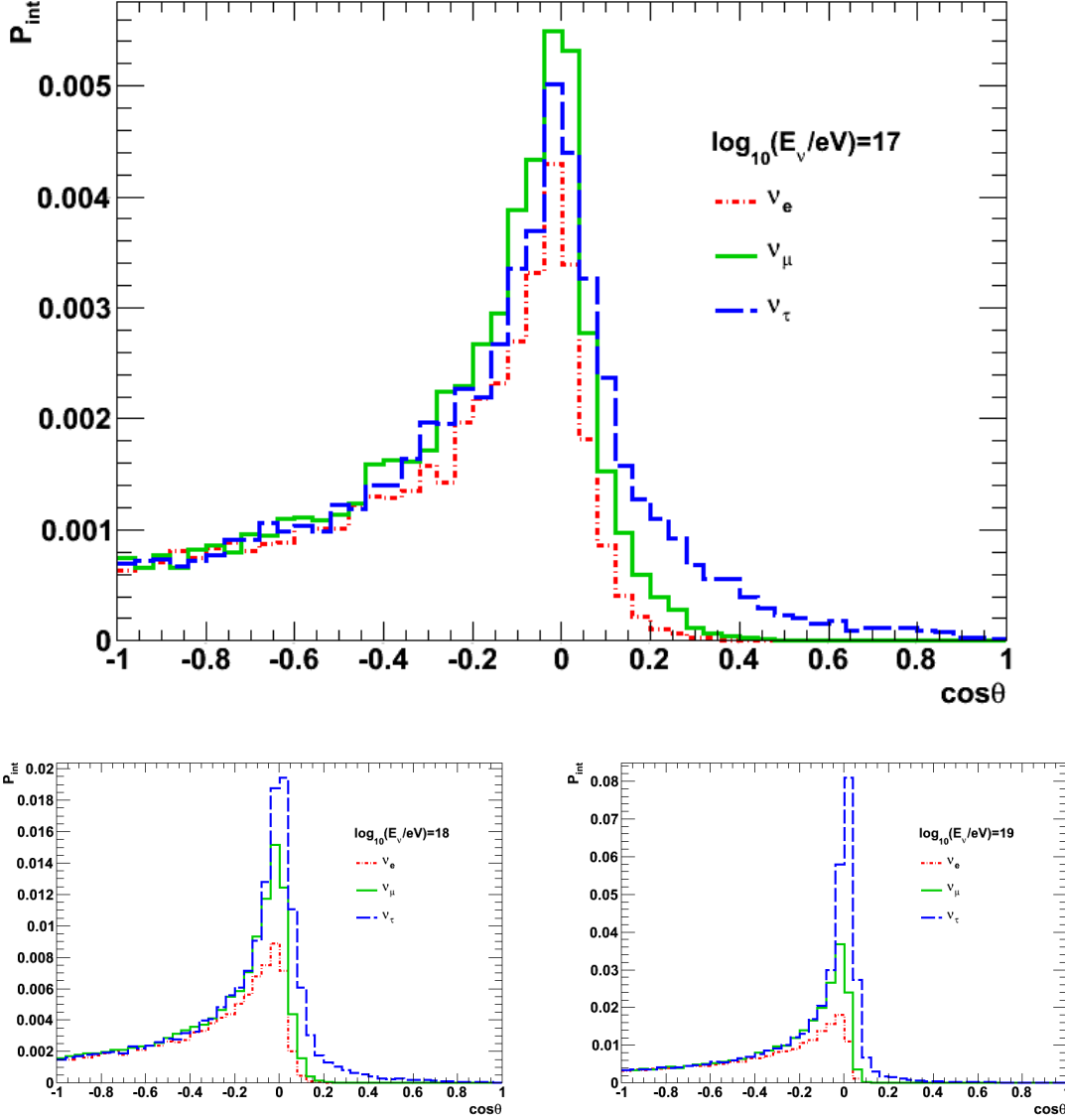


FIG. 3. Interaction probability P_{int} (see text for definition) as a function of the neutrino zenith direction $\cos\theta$ for ν_e (red dash-dotted), ν_μ (green solid), and ν_τ (blue dashed), and for initial energies $E_\nu = 10^{17}$ eV, 10^{18} eV and 10^{19} eV (from top to bottom panel).

emitted. For ν_μ , although there are plenty of electromagnetic showers produced by muon, these showers tend to have lower energies so that they are less likely to be detected by the sparse antenna array. This leads to lower detection efficiency of ν_μ , where the NC-induced hadronic showers account for about 80% of detected events for 1 EeV ν_μ . The situation is similar for ν_τ , where most hadronic showers from photonuclear interaction and electromag-

netic showers from pair production do not trigger detector efficiently while hadronic showers induced by NC interaction and τ decay ($\tau \rightarrow \nu_\tau + \text{hadrons}$) account for the most detected events. The convergence of efficiency for up-going ν_τ s of different initial energies results from the degradation of neutrino energy to few PeV by the regeneration process and NC interaction [53].

D. Event Direction Distribution and All-Sky Flavor Ratios of Events

Because the initial neutrino energy is sampled only with discrete values of equal logarithmic interval $\Delta \equiv \Delta \log_{10} E_\nu = 0.5$ in the calculation of P_{int} and ϵ_{det} , the direction distribution integrated over the j -th energy bin ranging from $\log_{10} E_j - \Delta/2$ to $\log_{10} E_j + \Delta/2$, is approximated by

$$\int D_\alpha(E_\nu, \cos \theta) dE_\nu \simeq \frac{1}{N} f_\alpha^E \left[\int_{E_j \times 10^{-\Delta/2}}^{E_j \times 10^{\Delta/2}} \Phi_\nu(E_\nu) dE_\nu \right] \times P_{\text{int},\alpha}(E_j, \cos \theta) \epsilon_{\text{det},\alpha}(E_j, \cos \theta), \quad (9)$$

with $\log_{10}(E_j/\text{eV}) = 17, 17.5, 18, 18.5, 19$ and 19.5 . The results assuming the ESS neutrino flux are plotted in Fig. 5, where the total area under the distributions for each flavor has been normalized to unity and the relative fraction contributed by each bin is also shown. It appears that EeV neutrinos contribute the most to the detected events for every flavor, because of the compromise between two competing effects: the decrease in the neutrino flux versus the increase in the detection efficiency with neutrino energy.

Finally, after summing over all distributions of different energy bins, the expected event direction distribution for each flavor $D_\alpha(\cos \theta)$ is obtained, which is shown in Fig. 6. We define the “all-sky” flavor ratio of detected events, $f_e : f_\mu : f_\tau$, as the event ratios among flavors after integrating over all zenith directions. If the ESS neutrino flux and incident flavor ratios at the Earth’s surface, $f_e^E : f_\mu^E : f_\tau^E = 1/3 : 1/3 : 1/3$, are assumed, we find that $f_e : f_\mu : f_\tau = 0.584 : 0.153 : 0.263$. The ν_e events account for the most portion because of their higher detection efficiency. The ν_τ events have a different shape compared to the other two flavors especially in the horizontal and up-going directions, primarily due to its special interaction probability (see the graph in the right panel, Fig. 6). These will be used to extract the flavor composition at the Earth’s surface (f^E s) in the next section. However, the resemblance between ν_e and ν_μ distributions will lead to the degeneracy in the flavor

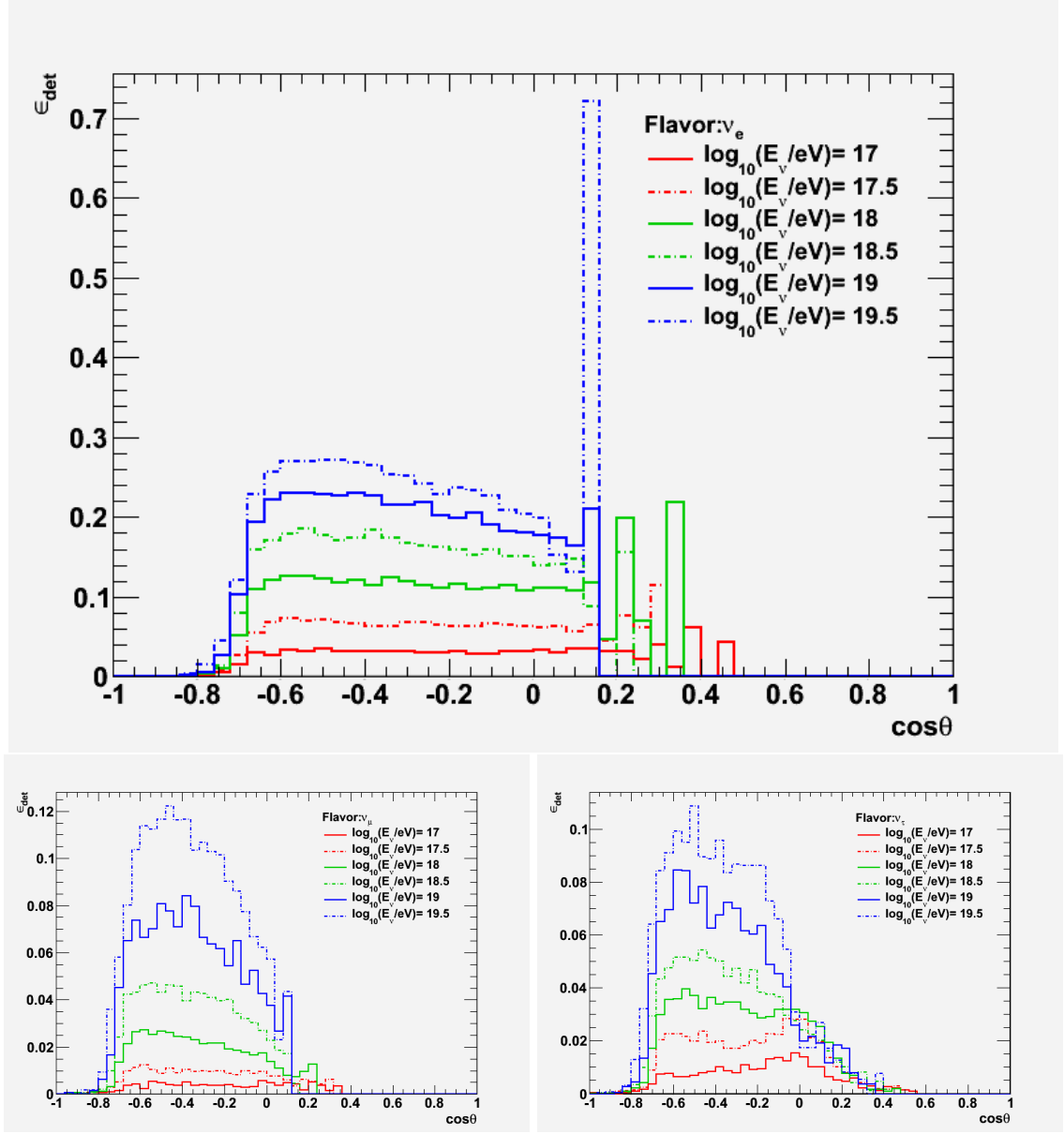


FIG. 4. The detection efficiency ϵ_{det} (see text for definition) as a function of neutrino zenithal direction $\cos \theta$ for initial energies $\log_{10}(E_\nu/\text{eV}) = 17$ (filled red squares; red solid line), 17.5 (open red; red dashed), 18 (filled green; green solid), 18.5 (open green; green dashed), 19 (filled blue; blue solid), 19.5 (open blue; blue dashed), and for flavor ν_e , ν_μ , and ν_τ (from top to bottom panel, respectively).

ratio extraction, and an extra constraint is required, for example, the ν_μ - ν_τ symmetry.

Once given reference incident flux ratios $f_{e,0}^E : f_{\mu,0}^E : f_{\tau,0}^E$ and the corresponding event ratios $f_{e,0} : f_{\mu,0} : f_{\tau,0}$, the all-sky event ratios $f_e : f_\mu : f_\tau$ for arbitrary incident flux ratios

$f_e^E : f_\mu^E : f_\tau^E$ can be deduced as

$$f_e = \frac{f_{e,0} f_e^E f_{\mu,0}^E f_{\tau,0}^E}{f_{e,0} f_e^E f_{\mu,0}^E f_{\tau,0}^E + f_{\mu,0} f_e^E f_{\mu,0}^E f_{\tau,0}^E + f_{\tau,0} f_e^E f_{\mu,0}^E f_{\tau,0}^E}, \quad (10)$$

and similarly for f_μ and f_τ . The conversion from f s to f^E s can be done by just interchanging f with f^E . The relation between ν_e event ratio f_e and flux ratio f_e^E is plotted in Fig. 7, assuming ESS neutrino flux.

IV. PSEUDO-OBSERVATION AND FLAVOR RATIO EXTRACTION

To investigate the discriminating power of flavor ratio reconstruction using event direction distribution, we generate pseudo-observation data from simulated events and fit the direction distribution to extract the flavor ratios, and repeat the processes to determine the statistical uncertainty of the extracted ratio. Results assuming different incident flux ratios, numbers of observed events, and angular resolution of detector are then presented.

A. Pseudo-Data Samples

The pseudo-data sample is prepared in three steps. First, detected neutrino events are randomly picked out one by one from the database of simulated events according to the expected direction distributions, $D_\alpha(\cos \theta)$, until any of direction bins of the database exhausts its events. Secondly, the zenith angle, θ , of each pick-up event is smeared by adding a Gaussian distributed random number with zero mean and the standard deviation $\Delta\theta$ equal to an assigned experimental angular error in reconstructed neutrino direction. Finally, this event collection is evenly divided into subsets, where each data set represents a hypothetical experimental data sample with a total number of detected events equal to N_{obs} and these data sets constitute a statistical ensemble.

In the following, N_{obs} varies from 50 to 500 with an increment of 50, while $\Delta\theta$ from 0° to 6° .

B. Fitting Pseudo-Data

To construct the fitting function for the hypothetical experimental data samples with angular resolution $\Delta\theta$, the expected direction distribution for each flavor α is convolved

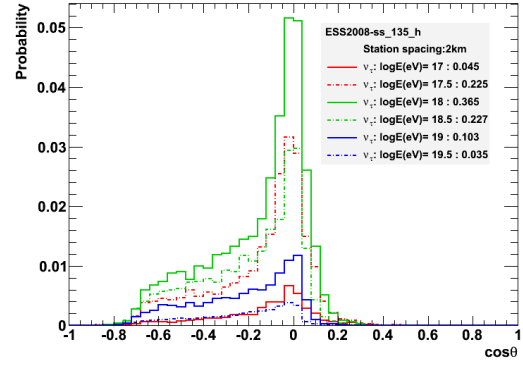
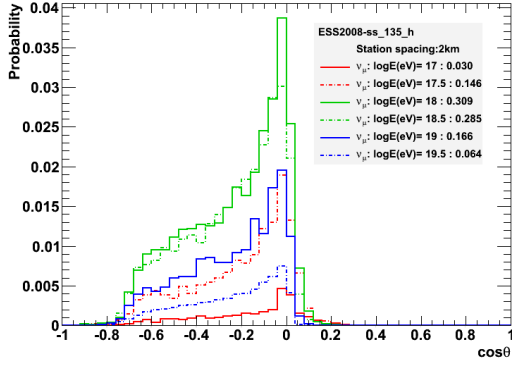
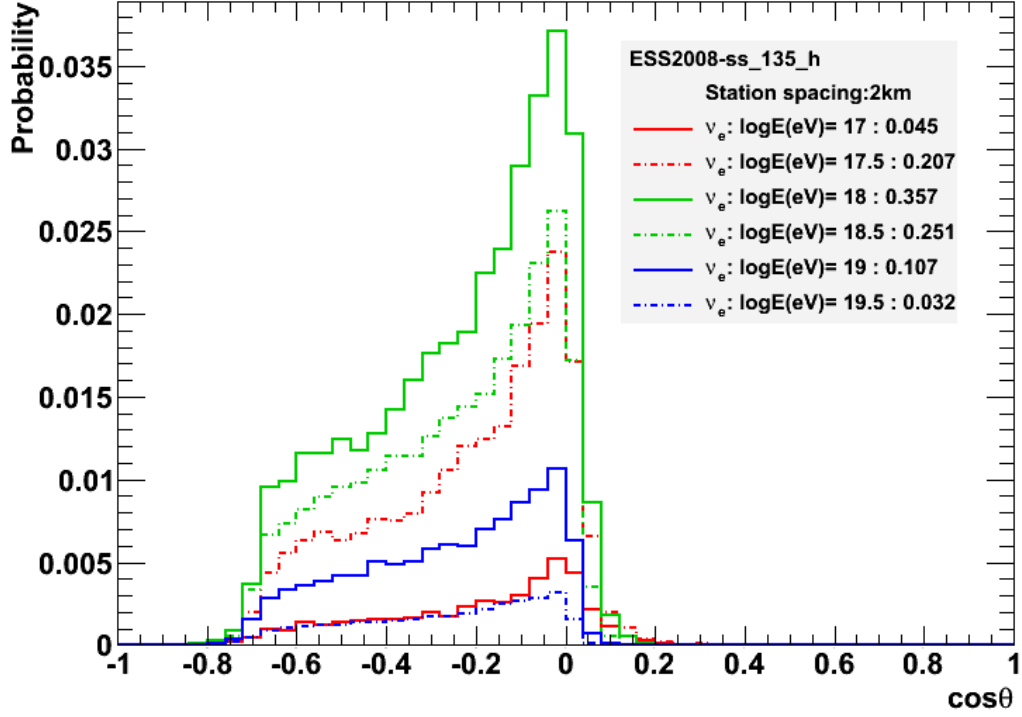


FIG. 5. The expected direction distribution integrated over the energy bins, $\log_{10}(E_j) \pm 0.25$, of initial neutrinos, where $\log_{10}(E_j/\text{eV}) = 17$ (red solid), 17.5 (red dashed), 18 (green solid), 18.5 (green dashed), 19 (blue solid), 19.5 (blue dashed), for flavor ν_e , ν_μ , and ν_τ (from top to bottom panel, respectively). The distributions have been normalized so that each one represents its fractional contribution to the corresponding flavor, and the relative fraction of each energy bin is listed in the legend. The ESS neutrino spectrum [72] is assumed.

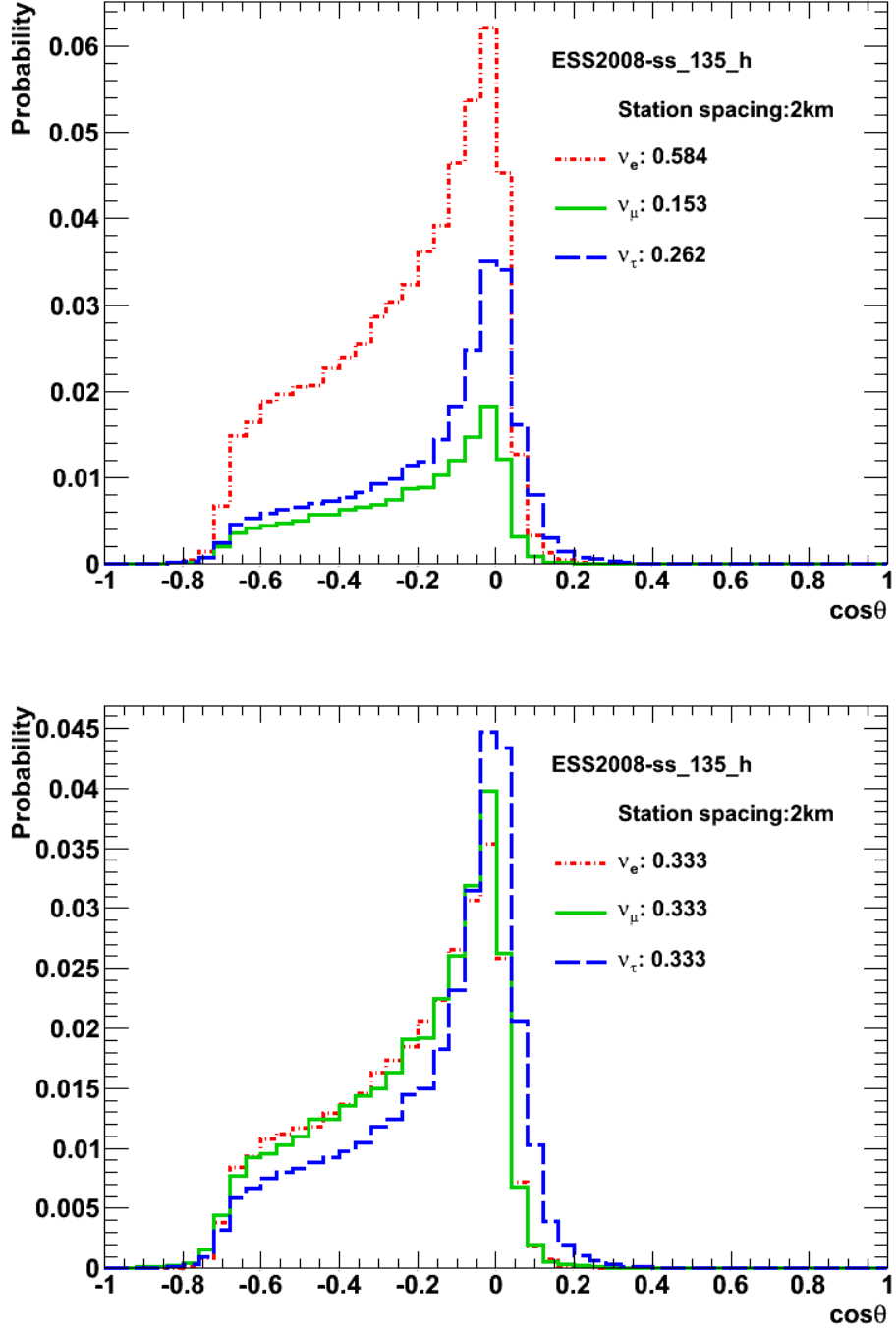


FIG. 6. Left panel: the expected direction distribution of detected neutrino events for each flavor α , $D_\alpha(\cos\theta)$, assuming the ESS flux [72] and the incident flux ratios $f_e^E : f_\mu^E : f_\tau^E = 1/3 : 1/3 : 1/3$. The all-sky flavor ratio of detected events $f_e : f_\mu : f_\tau$ is 0.584 : 0.153 : 0.263. Right panel: the expected direction distributions are scaled to equal fraction, 1/3 for each flavor, for comparing the curve shapes.

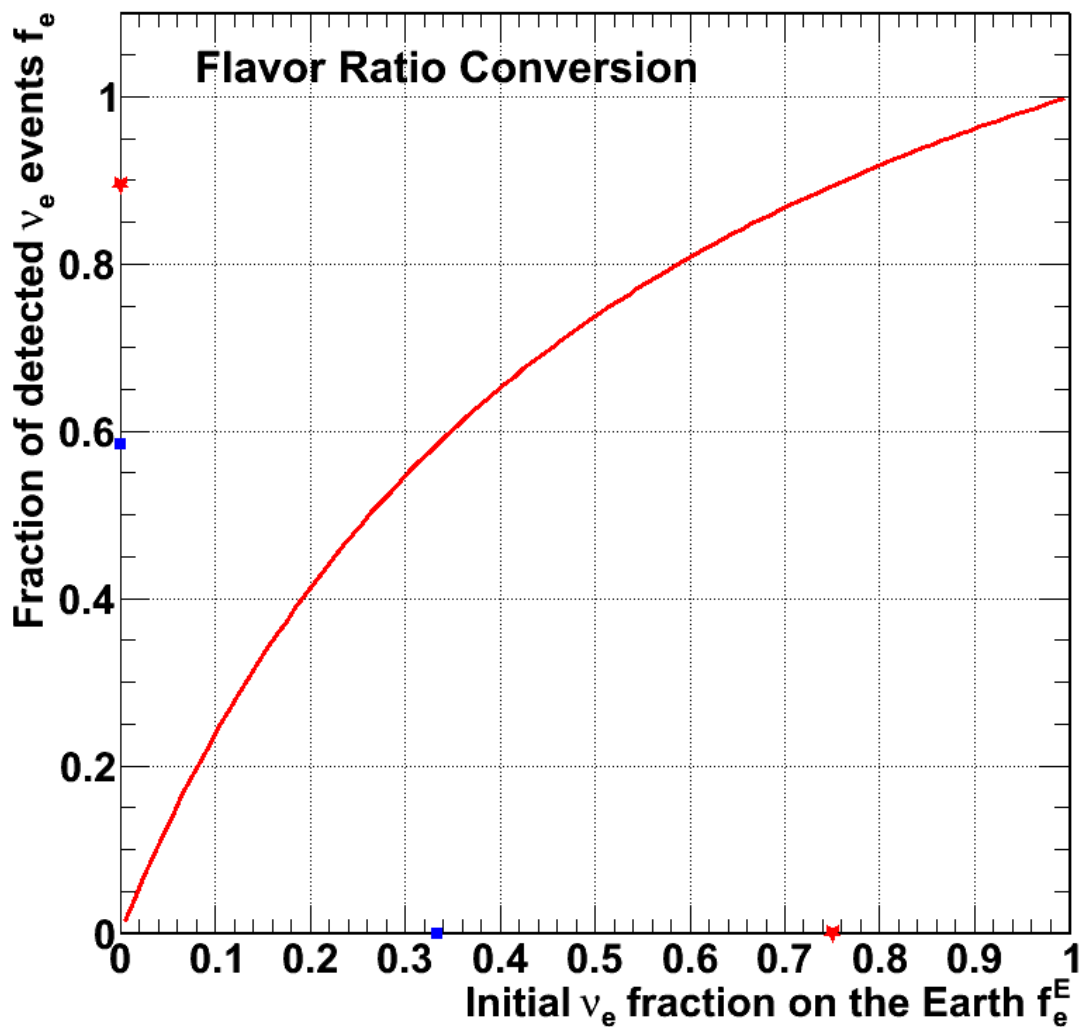


FIG. 7. The relation of the fraction of detected ν_e events, f_e versus the ν_e fraction of incident neutrino flux at the Earth's surface, f_e^E , where the ESS flux model [72] is assumed. The values for the standard case ($f_e^E : f_\mu^E : f_\tau^E = 1/3 : 1/3 : 1/3$, blue squares) and for the neutrino decay case ($0.75 : 0.125 : 0.125$, red stars) predicted in [19] are labeled on the axes.

with a Gaussian resolution function of standard deviation $\Delta\theta$ in θ space, $G(\theta'; \theta, (\Delta\theta)^2)$. The convolved distribution,

$$M_\alpha(\theta; \Delta\theta) \propto \int D_\alpha(\theta) G(\theta'; \theta, (\Delta\theta)^2) d\theta', \quad (11)$$

is then normalized so that

$$\int_{-1}^1 M_\alpha(\cos \theta; \Delta\theta) d \cos \theta = 1,$$

and the probability density function of event direction can be expressed as

$$P(\cos \theta; f_1, f_2, \Delta\theta) = f_1 M_e(\cos \theta) + (1 - f_1) \times [f_2 M_\mu(\cos \theta) + (1 - f_2) M_\tau(\cos \theta)], \quad (12)$$

where f_1 and f_2 are unknown fraction coefficients with values between zero and one. This expression ensures f_1 and f_2 are independent of each other, and they are simply related to the event flavor ratios by $f_e = f_1$, $f_\mu = (1 - f_1)f_2$, and $f_\tau = (1 - f_1)(1 - f_2)$.

The maximum likelihood estimation is applied for flavor ratio extraction. For an experimental data set with total events of N_{obs} , the likelihood function is defined as

$$L(\cos \theta_i; f_1, f_2, \Delta\theta) = \prod_{i=1}^{N_{\text{obs}}} P(\cos \theta_i; f_1, f_2, \Delta\theta), \quad (13)$$

where the subscript $i = 1, 2, \dots, N_{\text{obs}}$ stands for the i -th event. The true values of f_1 and f_2 are estimated by maximizing L , or equivalently minimize the negative log-likelihood (NLL),

$$-\ln L = -\sum_{i=1}^{N_{\text{obs}}} \ln P(\cos \theta_i; f_1, f_2, \Delta\theta). \quad (14)$$

We perform grid search in the parameter space to find the minimum of NLL and the associated best fit values of f_1 and f_2 , denoted as \hat{f}_1 (or \hat{f}_e) and \hat{f}_2 . A reconstruction is defined as failed if \hat{f}_1 or \hat{f}_2 is outside either boundary (zero or one). It may result from the insufficient statistics of events, or that the true value is near the boundary.²

However, an extra constraint is required to avoid multiple solutions arising from the similarity in distribution shape between ν_e and ν_μ , as pointed out in Section III D. The ν_μ - ν_τ symmetry, i.e. the incident ν_μ and ν_τ fluxes on the Earth are of equal amount ($f_\mu^E = f_\tau^E$), is imposed for this purpose. As a consequence, the value of f_2 is known and fixed, and there is only one parameter f_1 , the fraction of ν_e events, left to be fitted. In addition, for simplicity, we assume the observer knows the exact shape of neutrino spectrum; that is, the neutrino fluxes assumed for generating pseudo-data samples and fitting function are identical. The latter constraint will be relaxed in the next section and the mismatch in shape between real and expected spectra will introduce a systematic bias to the extracted ratio.

² When doing the fitting, \hat{f}_1 and \hat{f}_2 are permitted to have unphysical values.

C. Successful Probability and Flavor Ratio Resolution

After every data set in the ensemble has been fitted for a given N_{obs} and $\Delta\theta$, the successful probability of flavor ratio extraction is calculated as the number of successful data sets divided by the total number of data sets. The result is shown in Fig. 8, where the error bars indicate the uncertainties due to finite number of data sets and the shortest 68.3 % Bayesian credible intervals with uniform prior are used (see Appendix B, or [76] for details).³ The probability for initial ratio of 0 : 0.5 : 0.5 is not shown here because it is always around 50 % regardless of the number of observed events and angular resolution assumed.

In general, the successful probability increases with the number of observed events and a better angular resolution, as expected. The probability for the standard scenario (left panel) is greater than those for the decay scenarios (the one with normal hierarchy is shown in the right panel) since the latter have true ν_e ratios so closed to the boundaries that even small statistical fluctuation may result in failure.

For the standard scenario, the successful probability is greater than 70 % for $N_{\text{obs}} \geq 100$ and becomes over 90 % for $N_{\text{obs}} \geq 200$, if the angular resolution is within 6° . On the other hand, for the decay scenario with normal hierarchy, the probability is over 50 % once $N_{\text{obs}} \geq 100$. It was reported in Ref. [48] that fully deployed ARA is able to detect about 50 cosmogenic neutrinos in three years. Therefore our method is feasible for ARA to extract the flavor ratio of cosmogenic neutrinos.

We define the resolution of ν_e event fraction as the spread of all fitted values \hat{f}_e s in the ensemble with respect to the expected value $f_{e,\text{exp}}$,

$$R_{\pm}(N_{\text{obs}}, \Delta\theta) = \sqrt{\frac{1}{N_s - 1} \sum_{i=1}^{N_s} (\hat{f}_{e,i} - f_{e,\text{exp}})^2}, \quad (15)$$

where N_s is the number of hypothetical experimental data sets. If the fitting is not disturbed by the boundary cutoff, then the distribution of \hat{f}_e s asymptotically approaches a Gaussian distribution as the number of observed events N_{obs} increases, and therefore the resolution defined here is approximately 1σ uncertainty at 68 % confidence level. In order to reduce the effect induced by boundary cutoff, the resolution is calculated separately at both sides of the expected value and denoted as R_+ and R_- .

³ The concept of credible level is somewhat different from that of confidence level, because for the former our knowledge (the prior distribution) has been imposed.

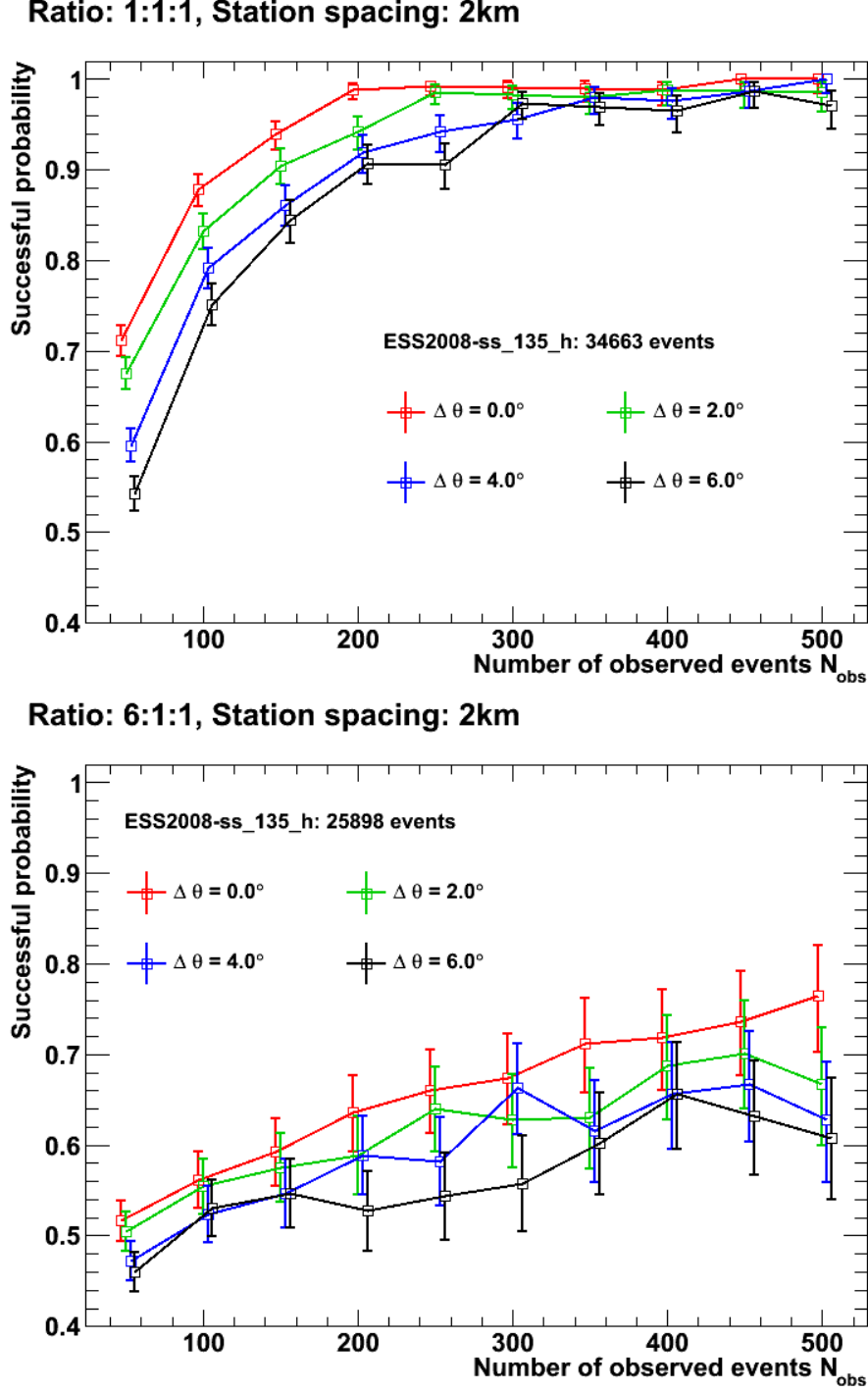


FIG. 8. The successful probability of flavor ratio extraction as a function of the number of observed events N_{obs} , assuming initial ratio of $1/3 : 1/3 : 1/3$ (left panel) and $0.75 : 0.125 : 0.125$ (right panel), where N_{obs} ranges from 50 to 500 with an interval of 50. Results for different angular resolutions are plotted: $\Delta\theta = 0^\circ$ (red), 2° (green), 4° (blue), 6° (black). The error bars, which result from the statistical uncertainty due to the finite number of data sets in the ensemble, represent the shortest 68.3% Bayesian credible intervals with uniform prior [76] (see Appendix B for details). Each curve is horizontally shifted slightly for clarity. The ESS neutrino flux [72] is assumed in both data generating and fitting, and the ν_μ - ν_τ symmetry is assumed in fitting so that the relative ratio between them is known and fixed.

The upper and lower bounds of ν_e event fraction, $f_{e,\text{exp}} \pm R_{\pm}$, are then transformed into the corresponding flavor ratios at the Earth's surface, $f_{e,\text{true}}^E \pm R_{\pm}^E$, according to Eqn. 10 (with f interchanging with f^E ; see also Fig. 7). The result for given N_{obs} and $\Delta\theta$ is shown in Fig. 9. The upper bounds for the initial ratio of 0.75 : 0.125 : 0.125 (blue error bars) are less meaningful because of the boundary cutoff. The resolution of ν_e ratio on the Earth R^E for given N_{obs} and $\Delta\theta$ is plotted in Fig. 10. The lower (upper) resolution is taken in the decay scenario with normal (inverted) hierarchy. The resolution in the standard scenario is asymmetric on both sides, with the upper resolution worse than the lower one typically by about 0.06, due to the nonlinear conversion relation between event ratios and flux ratios (Fig. 7), and hence the average value is taken.

To measure the separability of a flavor ratio predicted in a “fake” scenario from that extracted from a given “true” scenario, we define the discriminating power of flavor ratios as

$$\text{discriminating power} \equiv \frac{|f_{e,\text{model}} - f_{e,\text{exp}}|}{R}, \quad (16)$$

where $f_{e,\text{model}}$ is the ν_e event ratio expected by the scenario to be examined, and $f_{e,\text{exp}}$ and R are the expected ν_e event ratio and its resolution in the assumed true scenario, respectively. Note it is the ν_e “event” ratio and its resolution that are used in the definition because of the asymptotic normality of its distribution, whereas the distribution of ν_e flux ratio f_e^E is skew due to the nonlinear conversion relation. The discriminating powers assuming different true scenarios are shown in Fig. 11.

In Ref. [48], the ARA simulation result reports an angular resolution of neutrino direction about 6° . Under this circumstances, given 100 observed neutrino events, the successful probability is about 75 % in the standard scenario while about 50 % for both decay scenarios (black curves in Fig. 8 for the one with normal hierarchy). If the standard scenario is the real one, the discriminating power with respect to the decay scenario with normal (inverted) hierarchy will be at about 1.1σ (2.2σ) level. If the decay scenario with normal (inverted) hierarchy is real, then the discriminating power with respect to the standard scenario will be at about 0.8σ (1.4σ) level, while that to its inverted (normal) counterpart will be at about 2.4σ (2.2σ) level. Hence a preliminary constraint on the neutrino decay can be set by the method proposed in this article.

If the number of the observed events is doubled with the angular resolution fixed ($N_{\text{obs}} = 200$, $\Delta\theta = 6^\circ$), the successful probability rises to 90 % for the standard scenario while still

around 50 % for the decay ones; the discriminating power between the standard scenario and either of the decay scenario increases by about 0.2σ while by about 0.4σ between the decay scenarios. If the angular resolution is improved by 4° with the number of observed events fixed ($N_{\text{obs}} = 100$, $\Delta\theta = 2^\circ$), the successful probability becomes about 83 % for the standard scenario while about 55 % for the decay one with normal hierarchy; and the discriminating power between the standard scenario and the decay one with normal hierarchy increases by about 0.2σ , and by about 0.1σ for the decay one with inverted hierarchy to the other two while by about 0.4σ in the contrary case. The limit of our method with perfect angular resolution is also shown in Fig. 8, Fig. 10 and Fig. 11 (red lines).

To accumulate more neutrino events, apart from waiting for more neutrinos to come, one can increase the event rate by extending the antenna array. To achieve a higher angular resolution of neutrino direction, on the other hand, an antenna array with denser grid is required for a better imaging of the Cherenkov cone. Therefore the results presented here can serve as a reference for optimizing the future detector configuration of ARA or other similar observatories.

D. Systematic Uncertainty from Neutrino Spectrum

Fig. 12, Fig. 13, Fig. 14 and Fig. 15 show the results assuming the neutrino flux predicted in the “plausible” scenario in Ref. [14] (blue dashed curve in Fig. 2). The successful probability is lower than that assuming the ESS flux and the resolution is worse. Recall that the flux in Ref. [14] is steeper than the ESS one, i.e. there are more neutrino events with lower energies. Since the interaction probabilities between flavors are less distinct from each other at lower energies (see Fig. 3), the event direction distributions between flavors are harder to be distinguished.

If the observer does not exactly know the “real” neutrino spectrum (i.e., the one used to generate pseudo-data), then the mismatch in shape between the real and the expected (i.e., the one used to fit the data) spectra will introduce a systematic bias to the extracted ratio. We found that if the real spectrum is the plausible scenario in [14] while the spectrum expected by the observer is the ESS model, the bias is about 10 %. More specifically, the average fitted value drops from 0.58 to 0.53, because a steeper neutrino spectrum leads to more events coming from below the horizon due to the longer interaction length of neutrinos

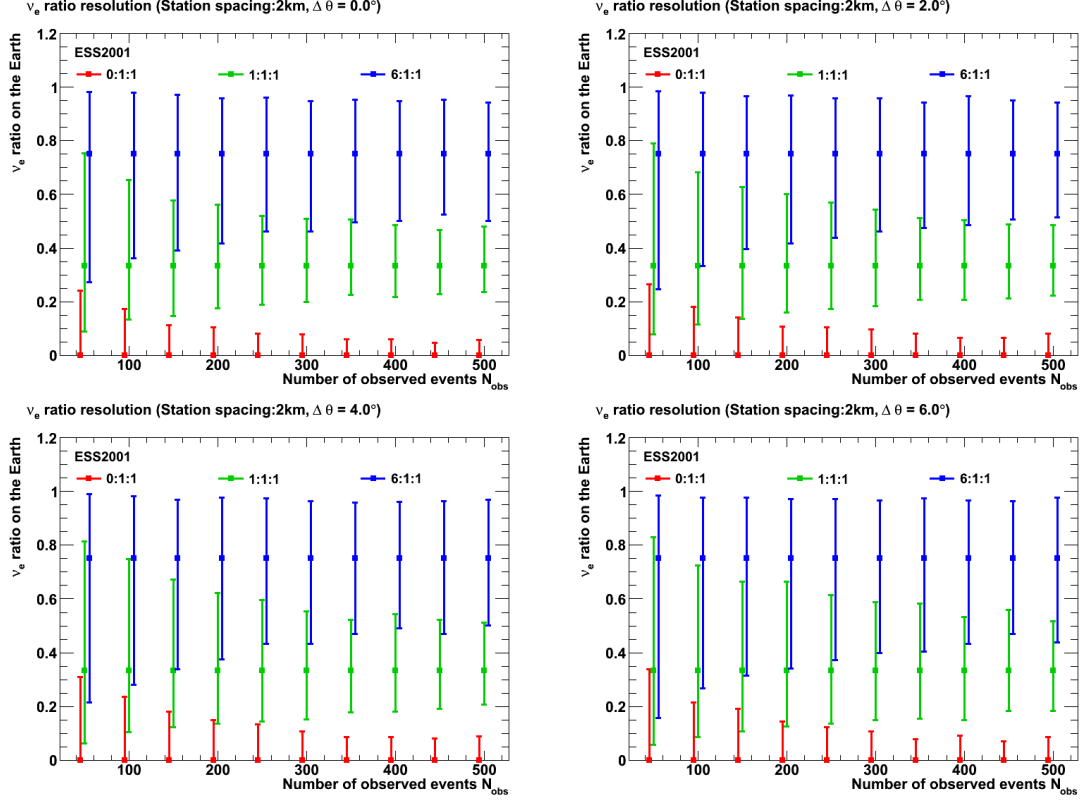


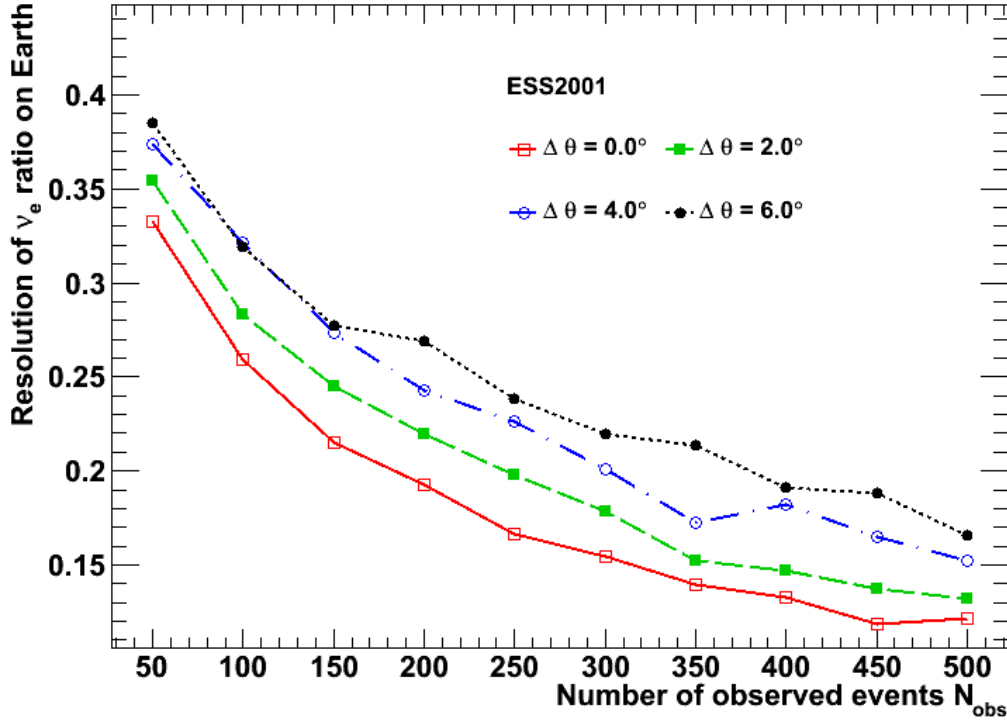
FIG. 9. The resolution of ν_e ratio at the Earth's surface plus the true ratio $f_e^E \pm R_{\pm}^E$ (see the text for definition) versus the number of observed events N_{obs} for initial ratios $f_e^E : f_{\mu}^E : f_{\tau}^E = 1/3 : 1/3 : 1/3$ (green, middle), $0.75 : 0.125 : 0.125$ (blue, top), and $0 : 0.5 : 0.5$ (red, bottom), where N_{obs} ranges from 50 to 500 with interval of 50. Results assuming different angular resolutions are shown: $\Delta\theta = 0^\circ$ (the perfect case, top left panel), 2° (top right), 4° (bottom left), and 6° (bottom right). The error bars for different initial ratios are horizontally shifted slightly for clarity. The ESS neutrino flux [72] is assumed in both data generating and fitting, and the ν_{μ} - ν_{τ} symmetry is assumed in fitting so that the relative ratio between them is known and fixed.

with lower energies. In order to fit the event distribution from the steeper spectrum with the flatter one, the ν_{τ} fraction has to be increased while the ν_e fraction decreased.

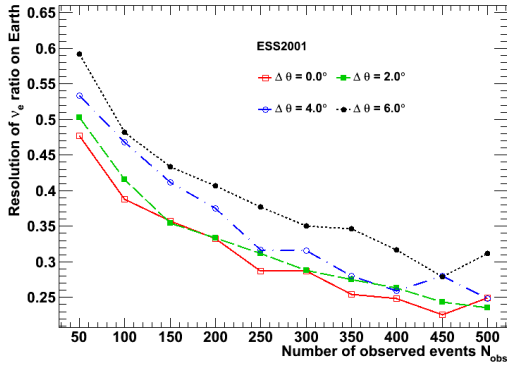
E. Systematic Uncertainty from Neutrino Cross Sections

The calculation of neutrino cross section in Ref. [61] reports an uncertainty of factor of $2^{\pm 1}$ at around 10^{20} eV. Therefore we vary the neutrino cross section by a factor of two,

Ratio: 1:1:1, Station spacing: 2km



Ratio: 6:1:1, Station spacing: 2km



Ratio: 0:1:1, Station spacing: 2km

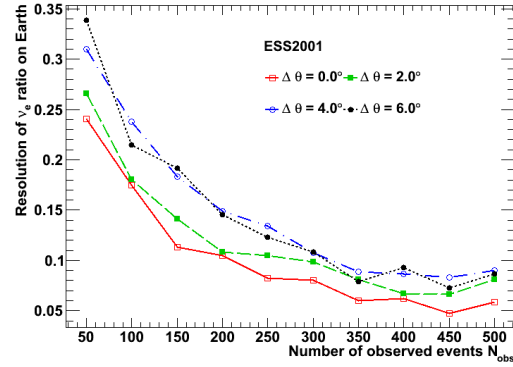


FIG. 10. The resolution of ν_e ratio at the Earth's surface R^E (see text for definition) versus the number of observed events N_{obs} for angular resolution of neutrino direction $\Delta\theta = 0^\circ$ (the perfect case, red solid line and open square), 2° (green dashed line and full square), 4° (blue dash-dotted line and open circle), and 6° (black dotted line and full circle). Results assuming initial flavor ratios of $1/3 : 1/3 : 1/3$ (top panel), $0.75 : 0.125 : 0.125$ (middle panel), and $0 : 0.5 : 0.5$ (bottom panel) are shown. The ESS neutrino flux [72] is assumed in both data generating and fitting, and the ν_μ - ν_τ symmetry is assumed in fitting so that the relative ratio between them is known and fixed.

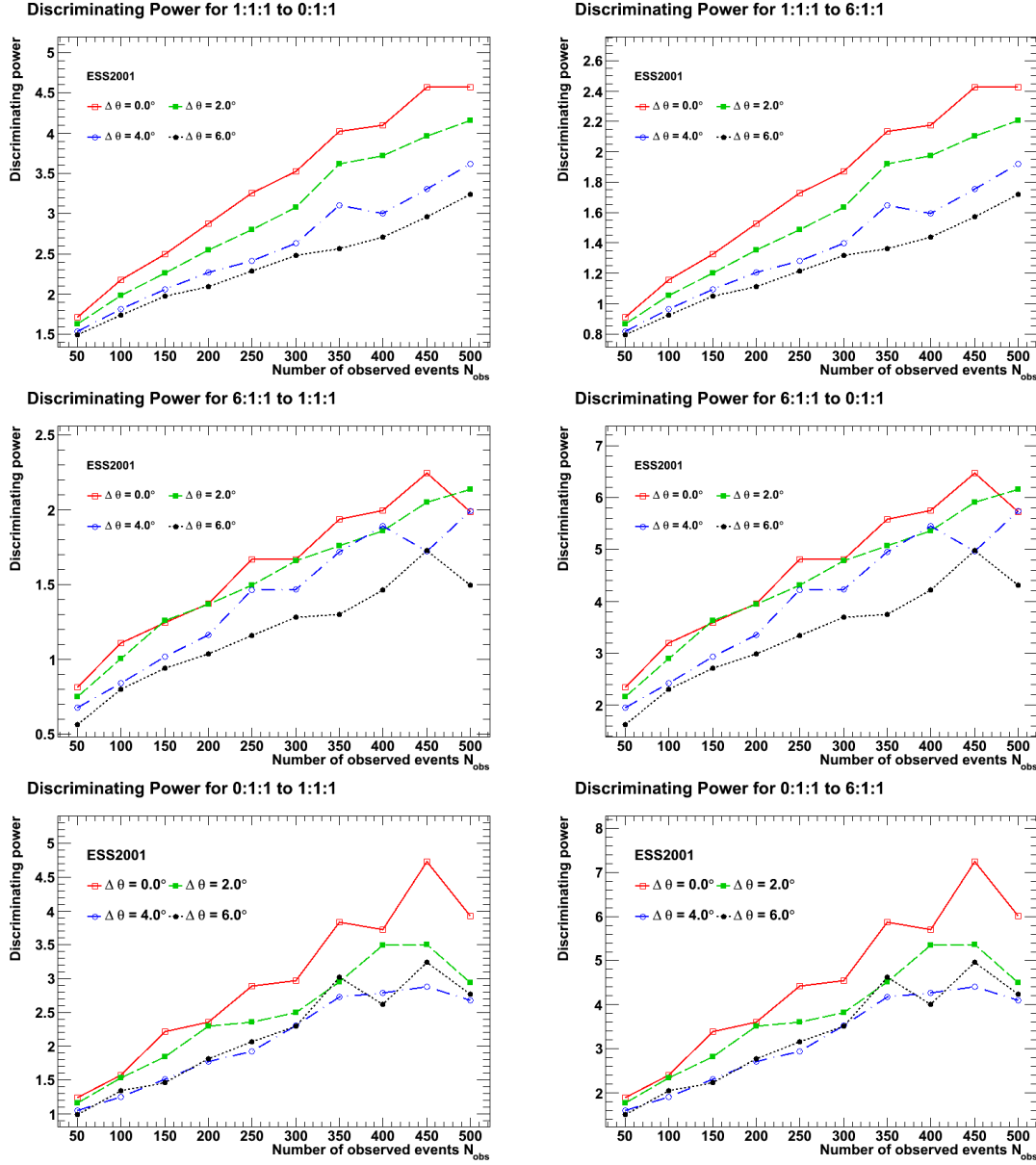
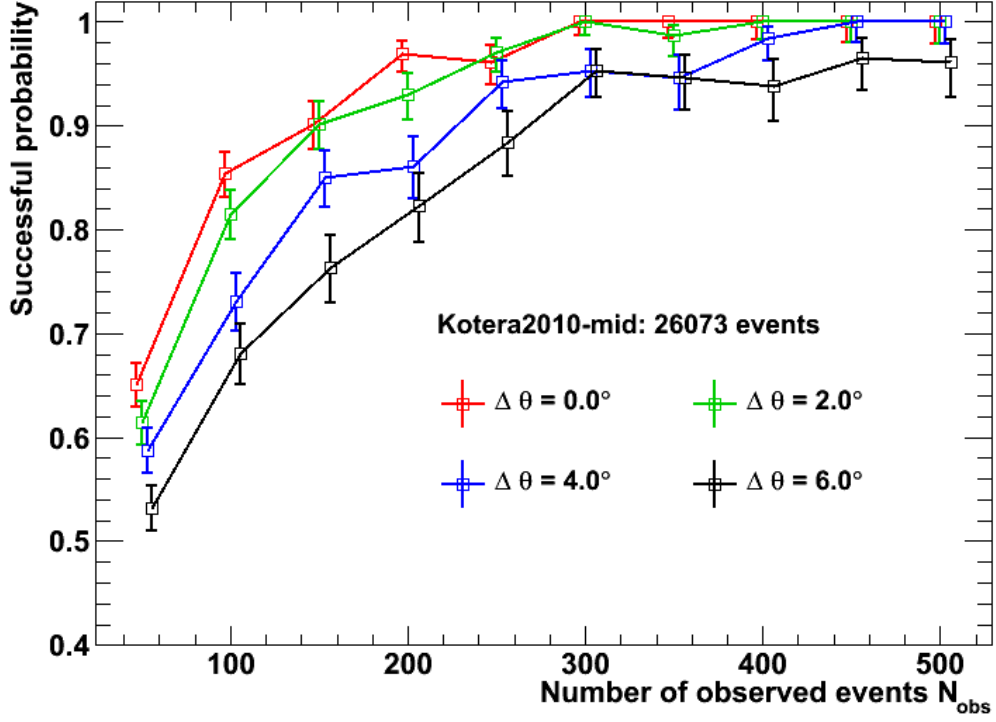


FIG. 11. The discriminating power of ν_e flavor ratio versus the number of observed events N_{obs} for angular resolution of neutrino direction $\Delta\theta = 0^\circ$ (the perfect case, red solid line and open square), 2° (green dashed line and full square), 4° (blue dash-dotted line and open circle), and 6° (black dotted line and full circle). Results assuming initial flavor ratios of $1/3 : 1/3 : 1/3$ (top panels), $0.75 : 0.125 : 0.125$ (middle panels), and $0 : 0.5 : 0.5$ (bottom panels) are shown. The ESS neutrino flux [72] is assumed in both data generating and fitting, and the ν_μ - ν_τ symmetry is assumed in fitting so that the relative ratio between them is known and fixed.

Ratio: 1:1:1, Station spacing: 2km



Ratio: 6:1:1, Station spacing: 2km

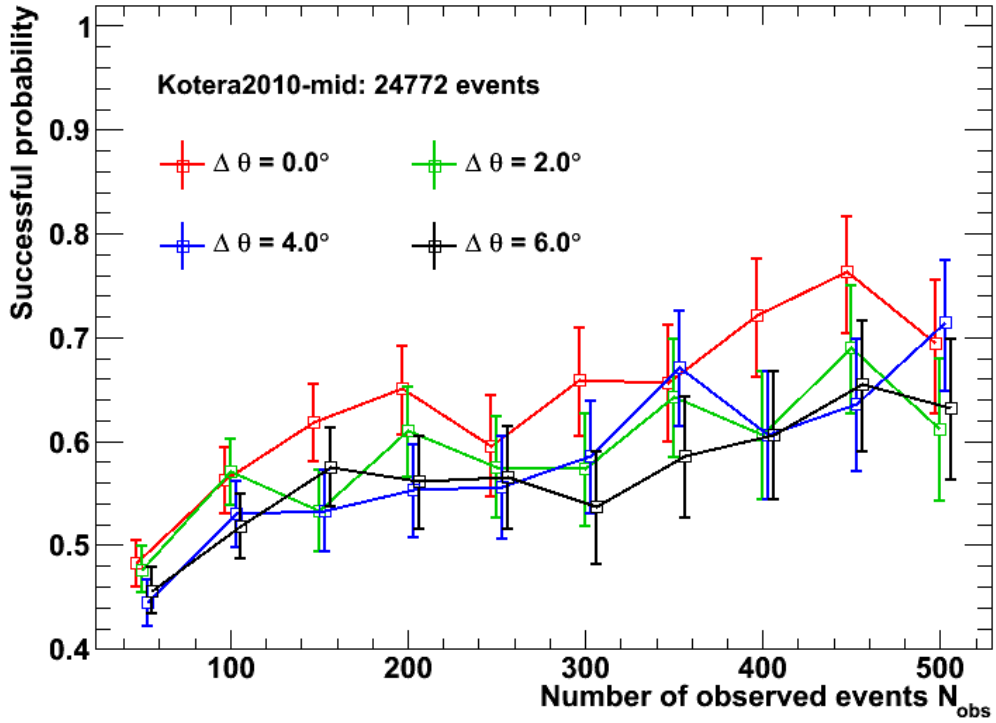


FIG. 12. The successful probability of flavor ratio extraction. Similar to Fig. 8 except that the assumed neutrino flux is the plausible scenario in Ref. [14] (blue dashed curve in Fig. 2).

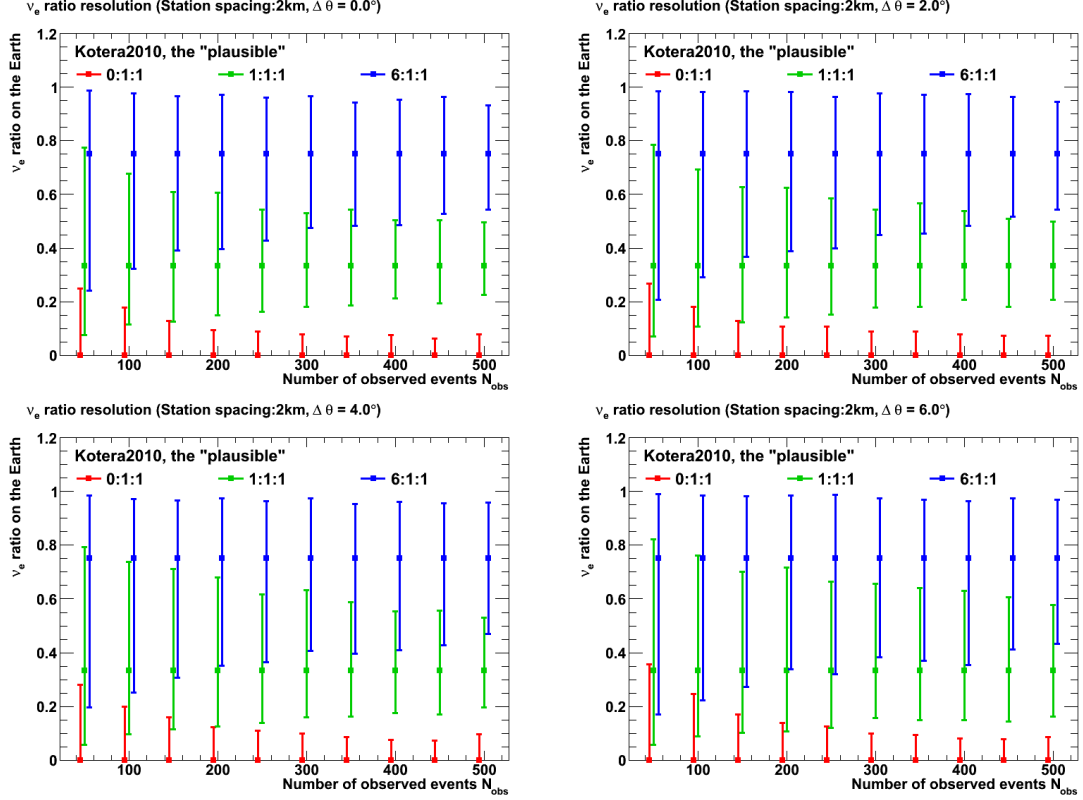


FIG. 13. The resolution of ν_e ratio on the Earth plus the true ratio. Similar to Fig. 9 except that the assumed neutrino flux is the plausible scenario in Ref. [14] (blue dashed curve in Fig. 2).

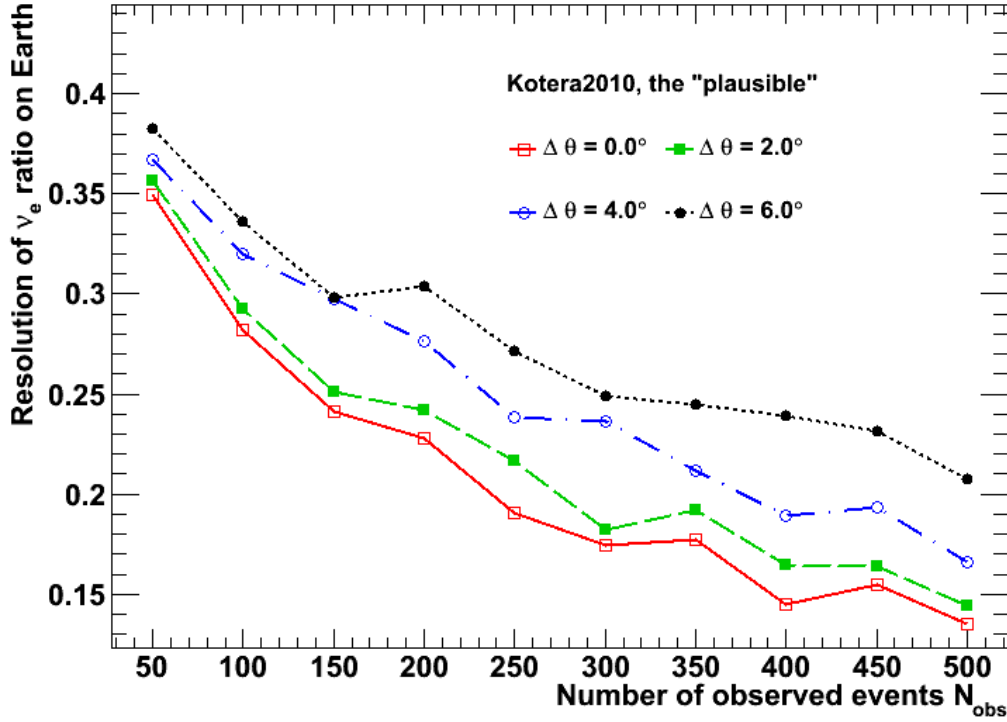
and the results are shown in Fig. 16, Fig. 17, Fig. 18 and Fig. 19.⁴ A larger neutrino cross section leads to lower successful probability and resolution of flavor ratio, because the shorter interaction length increase the fraction of events in the down-going directions where the interaction probabilities for different flavors are more or less the same, while makes the more distinct, up-going parts slightly decrease.

V. CONCLUSION AND FUTURE WORKS

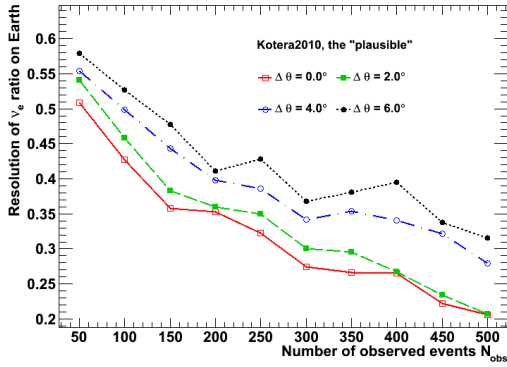
In summary, measuring the flavor ratio of UHE cosmic neutrinos can not only reveal the physical conditions at UHECR sources but also probe neutrino oscillation parameters and non-standard physics that might involve during the propagation. Neutrino observatories using the radio Cherenkov technique such as ARA [48], ARIANNA [49], and SaSA [50], are

⁴ For the results with neutrino cross section divided by two, the number of events in the ensemble are less than 5000, so I choose not to show them.

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Ratio: 6:1:1, Station spacing: 2km



Ratio: 0:1:1, Station spacing: 2km

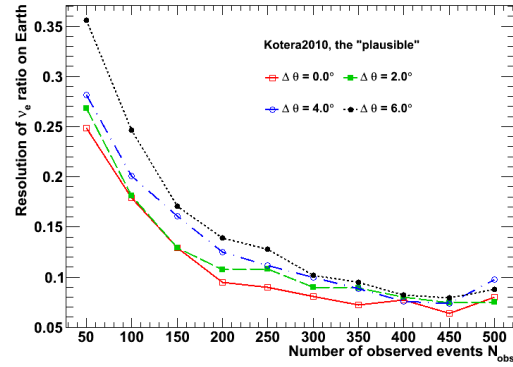


FIG. 14. The resolution of ν_e ratio at the Earth's surface versus the number of observed events. Similar to Fig. 10 except that the assumed neutrino flux is the plausible scenario in Ref. [14] (blue dashed curve in Fig. 2).

sensitive in the UHE regime and expected to accumulate of order of 10 to 100 cosmogenic neutrinos per year in the near future, and hence provide sufficient statistics for the flavor ratio identification.

In this work, the direction distribution of neutrino events is proposed to determine the flavor ratio. In order to investigate its feasibility, a simulation is constructed and the ex-

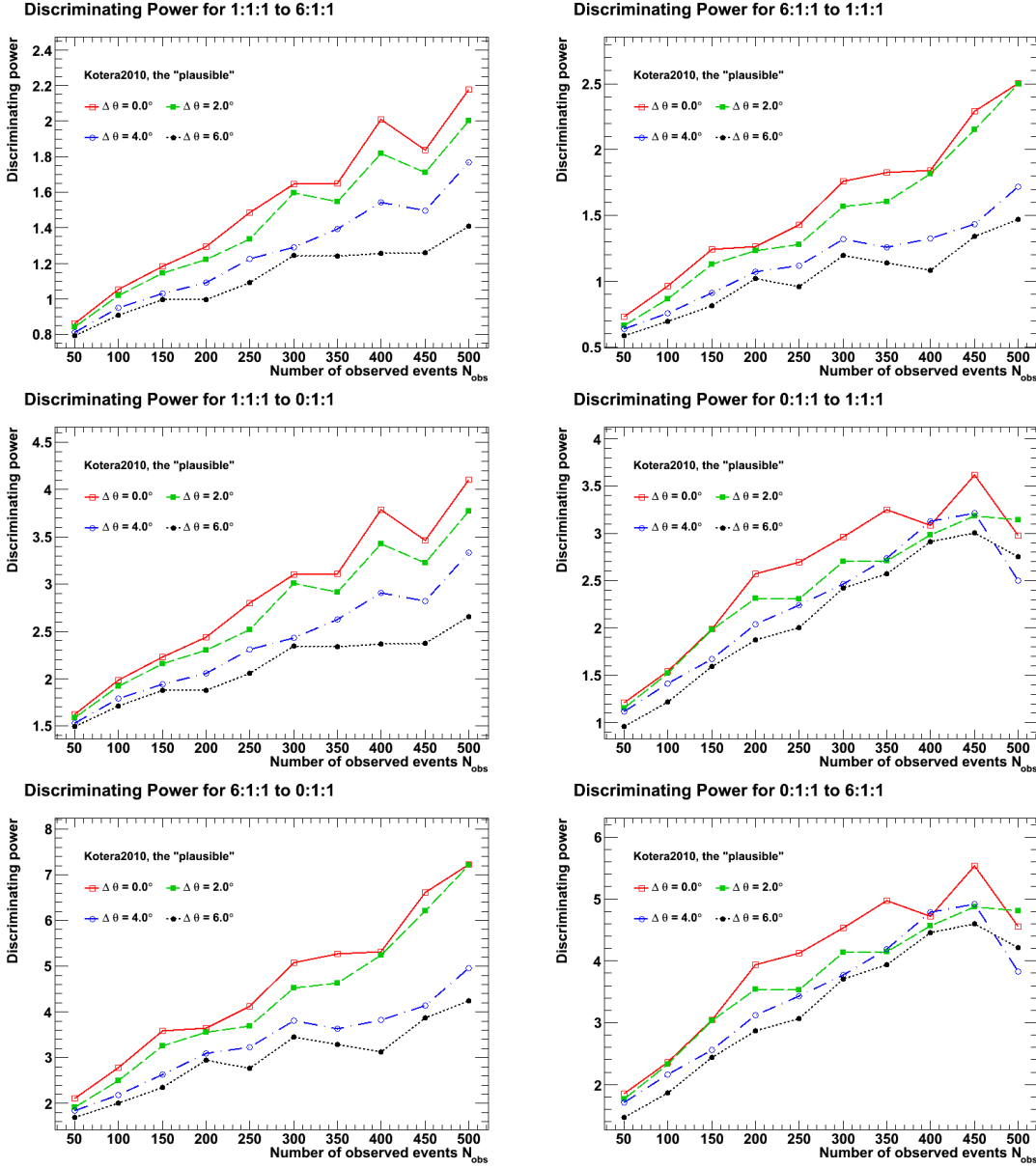
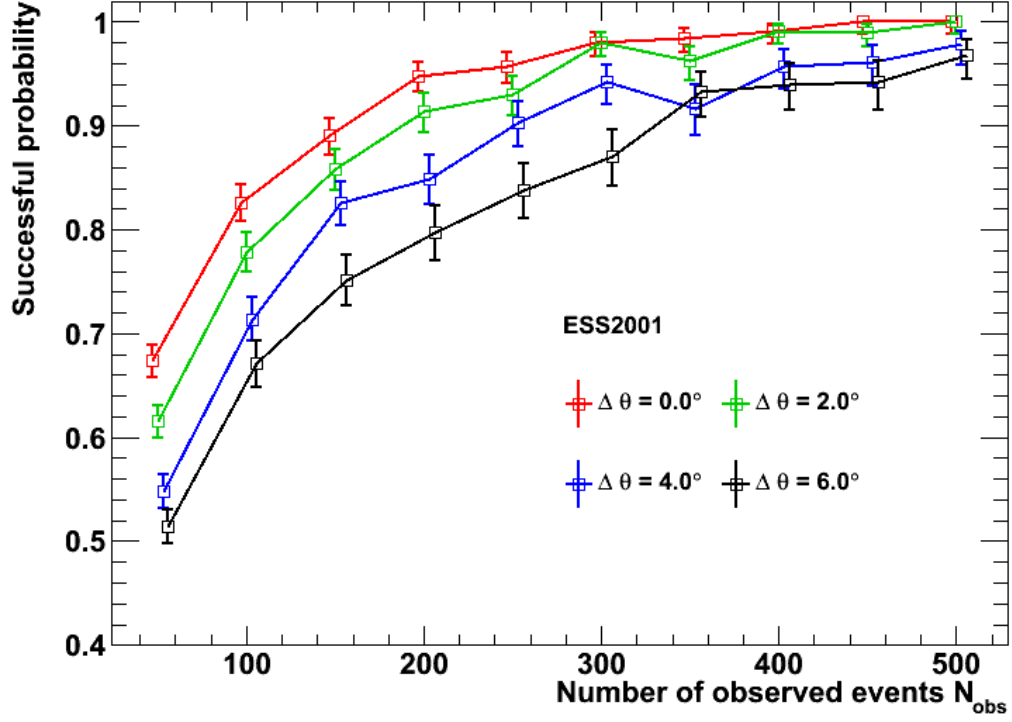


FIG. 15. The discriminating power of ν_e flavor ratio versus the number of observed events. Similar to Fig. 11 except that the assumed neutrino flux is the plausible scenario in Ref. [14] (blue dashed curve in Fig. 2).

pected distribution for ARA is derived. It is found that ν_τ distribution has a different shape from the other two, but the distributions for ν_e and ν_μ resemble each other. Therefore an additional constraint, the ν_μ - ν_τ symmetry, is imposed on the fitting of data distribution to avoid multiple solutions.

This method is proved to be feasible for ARA, e.g., for 100 events and an angular reso-

Ratio: 1:1:1, Station spacing: 2km



Ratio: 6:1:1, Station spacing: 2km

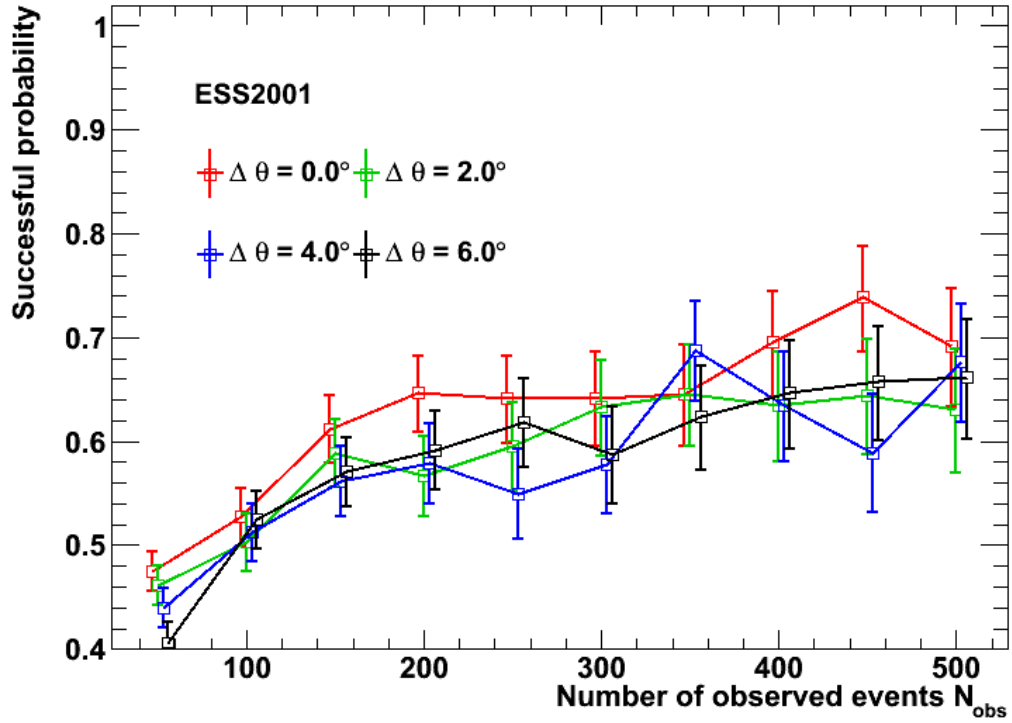


FIG. 16. The probability of successful flavor ratio reconstruction. Same as Fig. 8 except that the neutrino cross section is multiplied by a factor of two.

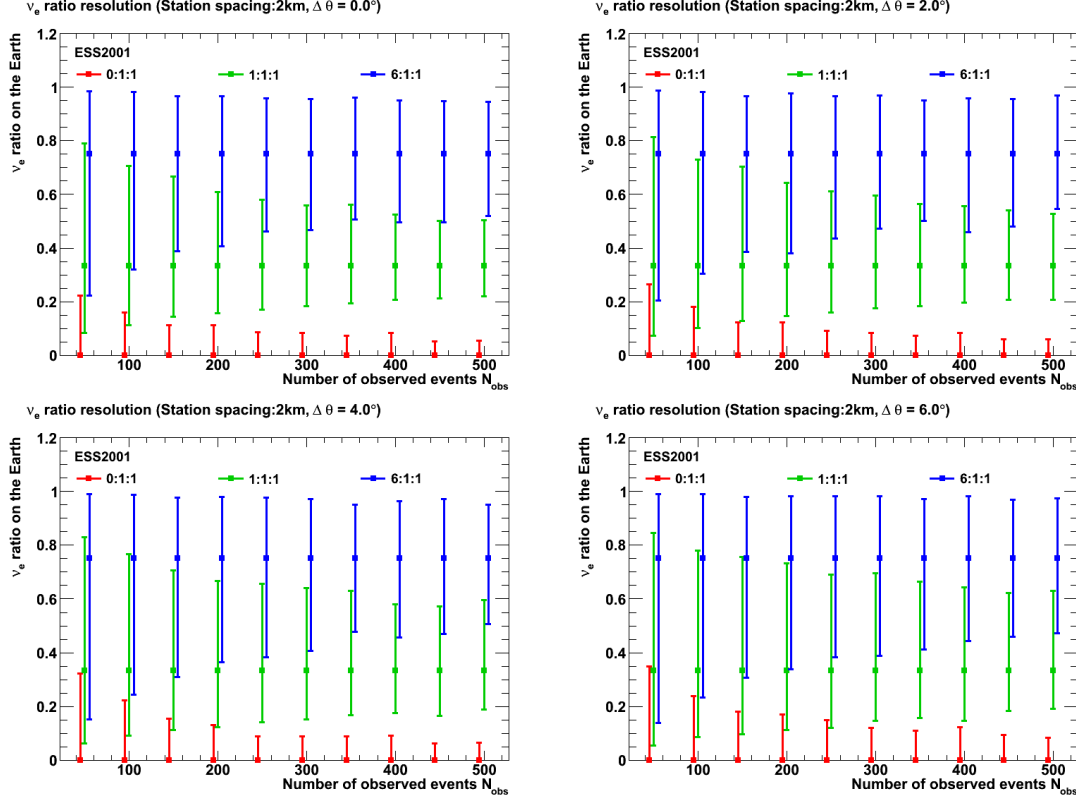


FIG. 17. The resolution of ν_e ratio on the Earth plus the true ratio. Same as Fig. 9 except that the neutrino cross section is multiplied by a factor of two.

lution of 6° , the successful probability is about 70 % in the standard scenario and over 50 % for the neutrino decay models considered here. This method is also able to set a preliminary constraint on the neutrino decay, e.g., given an angular resolution of 6° , it requires about 250 events to separate the standard scenario from the decay model with inverted hierarchy by about 1σ level. Therefore, the flavor resolution as a function of the number of observed events and the angular resolution of neutrino direction presented here can serve as a reference for optimizing the future configuration of ARA. Similar procedures can be done for other observatories.

However, we have not fully taken advantage of all the information possessed by neutrino events. For example, the spatial distribution and the energies of showers, which can be retrieved by incorporating the vertex reconstruction, can help to distinguish electron flavor from the other two; and the characteristics of radio signals may allow the classification of shower types and further the identification of the neutrino flavor event by event. In addition, the near-field effect of Cherenkov radiation has to be considered, since in some cases the

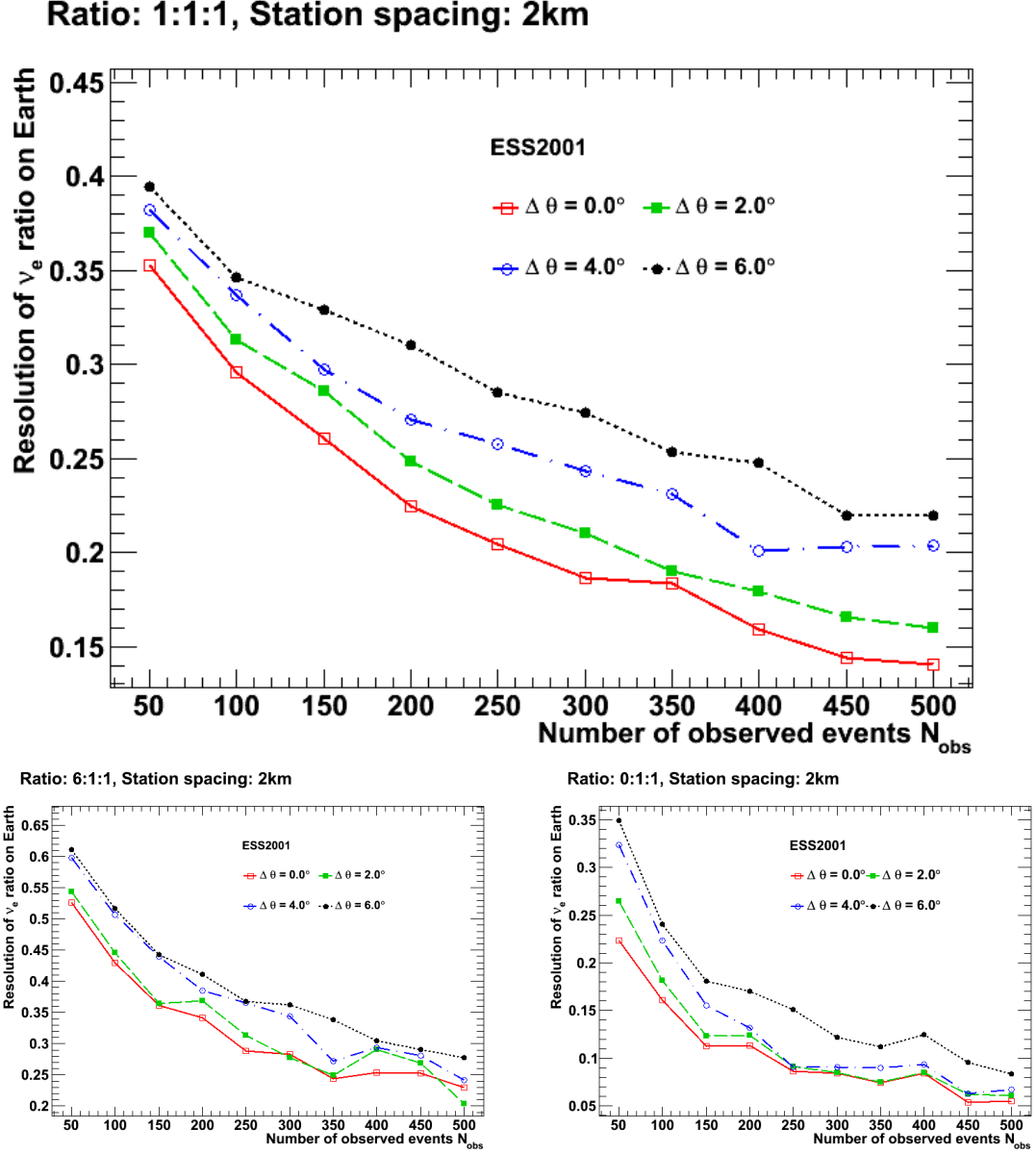


FIG. 18. The resolution of ν_e ratio on the Earth. Same as Fig. 10 except that the neutrino cross section is multiplied by a factor of two.

observation distances are comparable to the shower lengths and the far-field approximation applied here would underestimates both the signal strength and the angular width of the Cherenkov cone, as pointed out in Ref. [77]. The neutrino spectrum and the cross sections should also be incorporated into the fitting in order to reduce the systematic uncertainties. These will be included in the future simulation, and we expect to get a higher resolution of the neutrino flavor ratio.

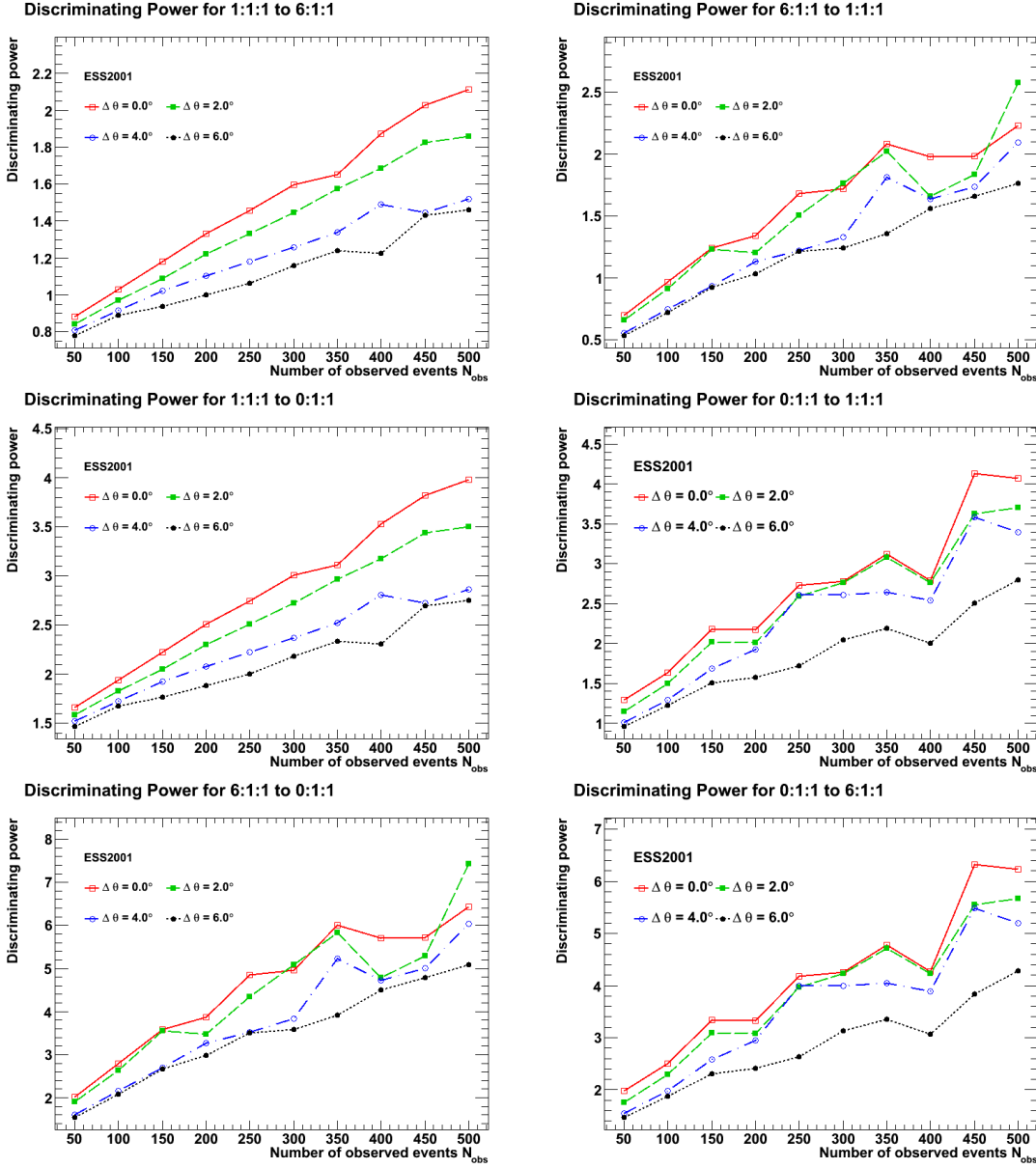


FIG. 19. The discriminating power of ν_e flavor ratio. Same as Fig. 11 except that the neutrino cross section is multiplied by a factor of two.

ACKNOWLEDGMENTS

We thank David Seckel and his coworkers of University of Delaware for sharing with us their simulation code SADE; our investigation would have been incomplete without this code. We also thank Guey-Lin Lin of National Chiao-Tung University for valuable discussions and suggestions. This work is supported by Taiwan National Science Council under Project No. NSC100-2119-M-002-025- and by US Department of Energy under Contract No. DE-

Appendix A: Calculation of Radio Cherenkov Spectrum

In SADE [?] ⁵, the calculation of Cherenkov radiation invokes the far-field approximation where the observation distance R is much longer than the longitudinal spread of the shower l_{sh} , $l_{\text{sh}}/R \ll 1$, and the radiation is assumed to be emitted from the point at the shower maximum. The frequency spectrum of Cherenkov radiation at an observation distance R and an observation angle θ with respect to the shower axis is parameterized as

$$R|\vec{E}(\nu, \delta, n, R)| = f_0 \nu \sin \theta \int dy f_y(y) e^{i\phi_y} \int dz f_z(z) e^{i\phi_z}, \quad (\text{A1})$$

where ν is the radiation frequency, $\delta = \theta - \theta_c$ the angular offset from the Cherenkov angle θ_c , n the index of refraction in the medium, f_0 the normalization factor, $\sin \theta$ the projection factor of the electric field to the line of sight, and y and z denote the transverse direction and the longitudinal direction along the shower axis, respectively. The two integrals containing the charge distributions of the shower f_y and f_z (with $\int dy f_y(y) = \int dz f_z(z) = 1$ over the shower) times the phase terms with the phases ϕ_y and ϕ_z account for the decoherence effect due to the spread of shower along the transverse and the longitudinal directions, respectively. If the emission from the entire shower is in-phase, the integrated value is equal to one, otherwise it is less than one. The integrals can be further parameterized into

$$G_y(\nu_y) \equiv \int dy f_y(y) e^{i\phi_y}, \quad (\text{A2})$$

$$G_z(\nu_z) \equiv \int dz f_z(z) e^{i\phi_z}, \quad (\text{A3})$$

with the scaled frequencies

$$\nu_y = a_y(\delta, n)\nu, \quad (\text{A4})$$

$$\nu_z = a_z(\delta, n)\nu, \quad (\text{A5})$$

and

$$a_y(\delta, n) = \frac{R_M}{R_{M,0}} \left(\cos \delta + \frac{\sin \delta}{\tan \theta_c} \right) \frac{\tan \theta_c}{\tan \theta_{c,0}}, \quad (\text{A6})$$

$$a_z(\delta, n) = \frac{l_{\text{sh}}}{l_{\text{sh},0}} \left(\sin \delta + \frac{1 - \cos \delta}{\tan \theta_c} \right) \frac{\tan \theta_c}{\tan \theta_{c,0}}, \quad (\text{A7})$$

⁵ This section is adapted from the technical document of SADE.

where R_M is the Moliere radius, the subscript 0 denotes the fiducial value used for scaling the spectrum to different medium or longitudinal spread of shower, and the angular terms arise from the phase difference. There is a similar discussion in Ref. [78].

The value of f_0 and the function forms of G_y and G_z are based on Ref. [67],

$$f_0 = \frac{2.53 \times 10^{-7}}{\nu_0 \sin \theta_{c,0}} \left(\frac{E_{\text{sh}}}{1 \text{ TeV}} \right), \quad (\text{A8})$$

$$G_y(\nu_y) = \frac{1}{1 + \left(\frac{\nu_y}{\nu_0} \right)^{1.44}}, \quad (\text{A9})$$

$$G_z(\nu_z) = \exp\left(-\frac{1}{2} \left(\frac{\nu_z}{a_{\nu, \text{AVZ}}} \right)^2\right), \quad (\text{A10})$$

with f_0 in unit of V MHz^{-1} , $\nu_0 = 1.15 \text{ GHz}$, $\theta_{c,0} = \arccos(1/1.78)$, E_{sh} the shower energy, and $a_{\nu, \text{AVZ}} = l_{\text{sh},0}(3.72^\circ/3.1 \text{ m})$. Note that the frequency ν of the lateral decoherence factor in the original parametrization of [67] has been replaced by ν_y , and the index of the exponential term in the longitudinal decoherence factor has a different expression.

Appendix B: Uncertainty Estimation of Efficiency

For the interaction probability and the detection efficiency defined in Sec. III, as well as the probability of successful flavor ratio extraction in Sec. IV, there is a selection efficiency ϵ introduced. Such selection efficiency is defined as the ratio of the number of entries satisfying certain condition, k , over the total number of entries, N :

$$\epsilon = \frac{k}{N},$$

with its value between zero and one. For all the three defined efficiencies, k is a random variable due to the underlying stochastic nature of particle interaction process, and whether an entry passes or not is independent of each other. Therefore the selection can be regarded as a binomial process with given total number of trials N and the unknown probability of success ϵ , and k follows the binomial distribution,

$$P(k|\epsilon, N) = \frac{N!}{k!(N-k)!} \epsilon^k (1-\epsilon)^{N-k}. \quad (\text{B1})$$

Due to the finite number of total entries, the estimated efficiency for given k and N is generally not the same as the true value, and therefore its deviation should also be disclosed.

Since it is shown that neither Poisson error nor binomial error can well represent the uncertainty when ϵ is near the boundaries [76], we instead follow the Bayesian approach presented in Ref. [76] to estimate the uncertainty.

According to Bayes' theorem, the probability distribution of ϵ for given k and N , or the posterior probability distribution of ϵ , is

$$P(\epsilon|k, N) \propto P(k|\epsilon, N)P(\epsilon), \quad (\text{B2})$$

where $P(\epsilon)$ is the prior probability distribution of ϵ , which includes our knowledge about the parameter ϵ before the data is considered. Because ϵ lies between zero and one, and there is no further knowledge about the true value, the uniform prior between zero and one is chosen,

$$P(\epsilon) = \begin{cases} 1, & \text{if } 0 \leq \epsilon \leq 1 \\ 0, & \text{otherwise} \end{cases}, \quad (\text{B3})$$

and then the posterior distribution after normalization becomes

$$P(\epsilon|k, N) = \frac{(N+1)!}{k!(N-k)!} \epsilon^k (1-\epsilon)^{N-k}, \quad (\text{B4})$$

which is a beta distribution with shape parameters of $(k+1)$ and $(N-k+1)$. The mode, i.e. the most likely value, is just k/N as expected by intuition, and the shortest credible intervals covering 68.3% of the posterior probability are chosen to represent the errors of efficiency in this article.

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TABLE II. Total number of simulated neutrino events arriving at the detection volume from the Earth's surface and having at least one interaction inside with the energy transfer larger than 1 PeV (the third column), as well as the number of detected events, for each flavor, initial energy, and station spacing of detector.

Flavor	Energy ($\log_{10}(E_\nu/\text{eV})$)	Total number of events	Number of detected events
ν_e	17.0	200000	5766
	17.5	200000	11884
	18.0	200000	20763
	18.5	200000	29291
	19.0	200000	36635
	19.5	200000	42079
ν_μ	17.0	200000	784
	17.5	300000	2783
	18.0	279400	5689
	18.5	200000	7246
	19.0	60000	3574
	19.5	100000	8704
ν_τ	17.0	200000	1750
	17.5	300000	5864
	18.0	276500	8146
	18.5	200000	7236
	19.0	80000	3876
	19.5	100000	6232

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